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IMAGERY AND ESTIMATION

'We have paid too little attention to the role of images
in mathematical understanding.' (Higginson, 1982)

Measurements frequently summon vivid
images. Could this have consequences for
the teaching of measurement estimation
skills to junior aged children?



CATHARINE LAWRENCE M.A.

A thesis for the degree of M. Phil in Mathematics Education
at the Open University

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ABSTRACT

Concentrating upon measurement, which the author believes may be an area rich in image building, the study starts with an historical overview of the naming of measure words. This examines their relationship to everyday experience and the colourful body of imagery that they evoke. She then turns to the contemporary and looks at present day adults and their measure-related imagery.

The study progresses with a literature survey of both the less recent and the current writings on imagery and moves from there to look at present educational practice and the place of measurement teaching within the National Curriculum. Drawing on her own experience and that of colleagues she chooses to examine a particular area of measurement teaching which can pose problems for children, measurement estimation.

The estimation process as understood both by modern classroom practitioners (and illustrated by main line teaching materials) and by educational researchers is examined in detail, from which a teaching plan for estimation is formulated. The work then concludes with some classroom experiments conducted to see if an increase in image building exercises can affect the accuracy of measurement estimation of the children involved.

The conclusions are tentative yet important and they are twofold in intention. On the one hand the work suggests to other teachers an area for development which may enhance their own teaching and so help their students, this being the use of image building exercises in estimation work. On the other hand, the work indicates related areas in which further research might usefully be directed.

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Introduction



IMAGERY AND ESTIMATION

'We have paid too little attention to the role of images in
mathematical understanding' (Higginson, 1982)

Measurements frequently summon vivid images. Could
this have consequences for the teaching of measurement
estimation skills to junior aged children?

INTRODUCTION

After fourteen years of teaching in the classroom I was first introduced to the term 'imagery' during a National Curriculum Mathematics course for Primary Mathematics Co-ordinators. The context was number skills and my mind buzzed with the excitement of the content. Surely this could have profound effects on that of my own and my colleagues' teaching skills. Were Higginson's words of 1982 true and had we really 'paid too little attention to the role of images in mathematical understanding' (p 239)?

Questions surged through my mind. Do children actually image; if so, how and when? If the answer were yes then does use of imagery vary with the type of maths they undertake? Is there any difference in the use of imagery between different ability pupils? If children were particularly encouraged to image and were given an image rich environment would they learn more easily? I was already using drawing as an integral and valuable part of my maths teaching, e.g. in multiplication work, and the value of this approach seemed linked to that of imagery. Could we as teachers deliberately tap into the importance of image building by constructing exercises which involved our pupils directly in this process?

In a notebook I began to jot down my own images and at the same time started to ask other adults about their own experiences of maths, and whether they had used images in their understanding. As the jottings grew what became increasingly apparent was that in the area of measurement images had played a powerful force in my friends and colleagues' maths vocabulary. It was an area of maths that appeared rich in image building. As I then turned to reading books on the history of measurement and measurements, these findings were confirmed. My interest became concentrated upon imagery in measurement and I began to apply my findings to my own teaching of measurement skills. In particular I

focussed upon measurement estimation which I found to be an area of work in which children often experienced difficulties.

I expanded my reading focus. From reading around the history of measures and measurement I next examined the past and present literature on imagery, from mid-nineteenth century to the present day. These writings have influenced both my thinking and my writing, although they did not in the main reflect directly upon the link between imagery and measures work. Further study centred on the role of measurement and estimation in today's mathematics curriculum as exemplified in the National Curriculum and in modern educational mathematics schemes. Finally an important focus of my reading has been to discover the findings of recent educational research into measurement estimation.

My general interest as to the role of imagery in measure related mathematics became more focussed as my research continued. I examined roles for imagery in the estimation process and then formulated my final conjecture that an increased use of imagery in estimation can increase accuracy. In pursuing these questions my practical work took several avenues. Included in my thesis are the findings from a small survey aimed at adults and their measure related images. I also worked with my own

classes, firstly in conducting a test in order to identify any weaknesses they might show in estimation skills, and later using imagery exercises in estimation. My final piece of practical work was conducted with a small group of children from another school in which I was looking for evidence to support my final conjecture.

In my writing I hope to show that images are frequently summoned by measurements, that this has been so historically, and that if this can be utilised in classroom teaching then children may experience greater success in their measurement estimation work.

Some understanding of my use of the words 'imagery' and 'images' will be needed in reading the earlier part of my work - a full discussion and definition of the terms coming towards the end of Chapter 3. As this later treatment will show I am taking imagery to encompass a wide range of internal experiences. So mental images may often be visual but they may also embrace use of the other senses; they may include auditory images or even the experiences of smell and taste. Imagery may be dynamic, and imaging may involve kinaesthetic experience. In contrast use of the word 'visualisation' within my text refers to mental images which are visual, pictures or diagrams being represented mentally. Its use is less general

than imagery. Visualisation may of course involve dynamic, moving picture images. I also refer to 'associations' of particular measures. In any situation we may have mental images triggered which act more as associations than as instruments for measurement and estimation work.

Chapter One

**Measure Words and the Rich Imagery they Invoke:
an Historical Overview**



Chapter One

Measure words and the rich imagery they evoke: an historical overview.

Looking back at the history of the naming of measure words reveals an image rich vocabulary. As I hope to show the words are found to be rooted in everyday experience and easily pictured language.

It is difficult to find evidence from very ancient civilisations such as those of the Amratians and Gerzeans. Nicholson, 1912, shows us that the Amratian civilisation had weights from as early as c. 3800 BC and the Gerzeans from c. 3300 BC, but the origins of the measures are not revealed. However by focussing back to more recent societies we *can* find the origin of certain measures. Some are found to be still familiar today.

Most of the measures I will look at are non-standardised, but some others are locally or even nationally standardised. Accurate standardised measurement has always been a specialist job which required education. In some civilisations it was a function for the priests but it was also an essential skill for merchants engaged in anything beyond the simplest commercial

transactions. Importantly, beneath the standard weights and measures there usually lingered easily visible names which were understandable by the uneducated.

The significance of moving to standardised measures is still a major issue in primary education. Measurement work with young children starts with non-standardised measures such as hand spans and shoe lengths and moves to the use of standardised measures and the reason for their use as the children mature.

For the sake of clarity I will look at early measures under five distinct categories, those of distance, area, weight, capacity and time.

Distance

Many civilisations used parts of the human body when quantifying length, including the Egyptians, the Greeks and the Romans. We find all of the following in use at different times:-

the foot, the cubit, the thumb, the finger's breadth, the palm, the span, the girth, the arm, the fathom, the hand and the digit. Even today horses are still measured by use of the 'hand'. The horse is measured from the base of the foreleg up to the top point of the shoulder.

The foot is perhaps the measure we are most familiar with ourselves. There were obviously some problems standardizing the measure! Under Charlemagne's rule in France the standard foot was taken from the King's own foot length!

For small amounts the barley corn was a common measurement. This was used in England in Medieval times. In 1324 three barleycorns became the measure for one inch. Another small measure was the nail. This was used for measuring cloth and meant the length of the nails across the four fingers on the hand. Longer lengths include the Roman double pace and the Greek stadion, originally the length of a race track. In England a longer length was the furlong (an eighth of a mile), which only disappeared with the demise of imperial units and remains today for measuring racecourses. The furlong took its name from a 'furrow length', a strip of land. Cricketers will be familiar with the chain, which now measures 22 yards. Originally it consisted of a chain of metal comprising 100 links.

These are only a few examples but they show that distance measures were often those which could summon an immediate visual image. No one would have trouble in thinking in terms of a 'span' or a 'cubit', since they had them constantly before them. (The 'span' being the width of their hand when

stretched and the 'cubit' the length from elbow to the tip of the index finger). The 'furrow length' would have been an everyday sight to medieval man, as he laboured on the land. Barley corns were a commonly used grain.

Area

Nicholson, 1912, shows us that land measures were reckoned in the following ways:

The first measures of land were seed-measures. . . . Then came the estimation of land by the amount of ploughing or sometimes of hand-digging, that could be done in a day, and by the extent that could be cultivated with a pair of oxen. (p 65)

Nicholson gives many examples, most of them from different cultures. Thus the 'estree' , the 'ouvree' and the 'journal' from France, or the 'moggio' and 'giornata' from Italy. In England we find the measure of the 'Daieswork', taken from a day's labour by hand. Then there is the acre - originally just the name for a stretch of land that could be tilled in one day, perhaps from the German 'acker' or else the Greek word 'agros' now meaning field. There was also the 'oxgang' unit used by the Saxons. Nicholson again:

The land that a boor with a yoke of oxen could keep in husbandry; about 7 acres of arable, about 30 acres including wood and pasture. (p 66)

Klein, 1975, points us to the French 'ouvree hommee', a measure used in the cultivation of vineyards, and the German measure 'morgen', meaning 'morning'.

One interesting area measure is unique to England. It was another Anglo Saxon term, the 'trithing', and according to Sally Duggan, 1993, meant 'one-third of a county. The Ridings of Yorkshire are a corruption of this term.' (p 40)

Once again these measures, like those of distance, are immediately visual and would have been meaningful in terms of images to the people using them.

Weight

Very few non-standardised units of weight are to be found. This is probably because weights were standardised very early by communities and civilisations for use in trade and commerce. Indeed 'The balance was the first scientific instrument to be invented, and is at least 7000 years old.'

(p 44), Duggan.

Klein, 1975, writes:

The first units of weight, . . . were often based on botanical objects, particularly seeds, the least variable identifiable parts of plants. (p82)

So grains of various sorts were used: carats, from the coral tree, wheat corns, and a measure known simply as a grain. Another early weight found by Klein is the clove. He believes this came from the Latin word for a nail, 'clavus'.

Looking further afield he finds some intriguing weights. There is the 'barrel' for herring and the 'coffee sack', which speaks for itself! Nearer to home we find the 'stone'. Although now standardised the original stone was based on real stones and therefore varied in different parts of the country.

Weights may have been standardised early for the purposes of trade but it is still possible to see in those that can be traced to primitive origins, that these origins are distinctly visual and experienced.

Capacity

Early measures of capacity and volume are fascinating. Small amounts were measured in 'mouthfuls'. Then we find the 'hogshead' for storing liquids. 'Barrels' are used again, for beer in particular. Klein, 1988, mentions the kilderkin:-

Centuries ago the half barrel was called the kilderkin, probably a corruption of the Flemish kinderkin, which in turn seems linked with the German diminutive Kinderchen, or 'little children'. (p 43)

Timber was measured in 'loads', which was the amount of wood which could be loaded onto a cart, and also in 'cords'.

The Romans used the measure of a 'spoonsful' for dry and liquid capacity measure. Interestingly even as recently as this century recipe books are found which measure in 'spoons' or 'spoonfuls'. 'Teacupfuls' were also used in many recipes. In the *Olio Cookery Book*, published in the 1920's, we find the following:-

HIGH CHURCH PUDDING

1 teacupful suet

1 teacupful breadcrumbs

teacupful flour

1 teacupful jam

1 teacupful milk

1 teaspoonful sugar

1 teaspoonful carbonate of soda

Steam two or three hours, (p 47)

Indeed it is not uncommon nowadays to find people still choosing to use cups and spoons as measures in their cooking preparations.

The Chinese used sounds to help give them accurate measures of liquid. They then used words which meant something to both the senses of sight and sound.

Containers for grain and wine were defined not only by weight but also by sound: given a uniform shape and weight, only a correctly-filled container will sound the right pitch when struck. In old Chinese the same word was used for wine bowl, grain measure and bell. (p 54) Duggan.

Time

Our terms for the measurement of time are based for the most part on astronomical features. Before the development of artificial light our ancestors were acutely aware of the sun and the phases of the moon, and their days focussed around them. As Klein, 1975, points out, we get our word 'month' from the original English word for 'moon', which was 'mona'. This of course would be the lunar month rather than the calendar month. Calendar months were introduced in more sophisticated times and vary in duration. There have been many different ways in which the months have been named and altered but for the purpose of this study it is interesting to see how the French named their new calendar in 1792.

Because the new calendar was supposed to symbolise the dawn of a new republican era, every reminder of Christianity was removed. . . .

Instead, the months were given names to suit the time of year in which they fell. . . . The first month which, predictably, began on 22 September, the day

the Republic was founded, was to be called Venemiaire, after the vendage, or wine harvest. This was followed by Brumaire, the misty month, Frimaire, the cold month, and so on. (ps 69, 71) Duggan.

The interest here lies in the fact that the names chosen deliberately appealed to imagery and visual and sensual memory.

The word 'day' means that unit of time which it takes for the earth to fully rotate. Civilisations had different chosen times for the start of the day whether it was dawn, dusk or midnight. The term 'day' is therefore defined by reference to the earth's movement and cannot be defined in any other terms, unlike our present day 'hours' and 'seconds' which are defined by reference to other time measures. So an 'hour' is one twenty fourth of a day and a 'second' is one sixtieth of a minute.

The 'year' is also unique in its meaning, being the full duration of the earth's rotation about the sun. Like days and lunar months its meaning is fixed in experience, and can be instantly understood. Years needed some reference point from which to be measured. From the fourth century for example the Greeks named their years with reference to the Olympic Games. Thus Dilke, 1989:-

Since the games started in 776 BC, that year counted as 01. (Olympiad) 1.1, 01.1.2. being 775; 01.2.1 (the year of the second games) was 772 BC, and so on. (p 40)

The Christian calendar was taken from the year of Christ's birth, although as Dilke points out it misfires by a few years.

What does this historical overview show us? Most importantly it shows that measure words often evoked rich imagery. This was no matter of chance. Originally measures were chosen because they were based on every day objects or events, commonplace to the contemporary society. Therefore those using the measures would have been able to summon meaningful images of them, even when not working with them directly, for example when in a discussion or during estimation. But if it is true historically that measures often summon vivid images, then what of present day adults and their measure related imagery?

Chapter Two

Present Day Adults and their Measure-related Imagery



Chapter 2

Present day adults and their measure-related imagery

"When I have to work out how far 100 yards is, I always think of the distance from the church to the pub in the village where I lived as a child."

"To me 1 pint means the amount in our white jug on the table at tea time."

"One dozen is just a tray of eggs."

"I think of a gallon as a large bucket with 8 pints of water from a milk bottle being poured into it."

"The months of the year are in a circle. They are grouped close together at first, starting with January on the top left hand side. When you reach July there is a break. August lasts for some distance on the circumference and then the months slowly reach December at the bottom of the circle. This month occupies almost one third of the circle before joining January with an arrow."

These and many other lively descriptions were given to me by adults when asked to talk about the way in which they think of measure terms, or work out measurement problems. Most adults showed that they had acquired a unique body of imagery to help them tackle common measurement tasks. This added fuel to my growing conjecture that - MEASURES SUMMON VIVID IMAGES.

This being so I conducted a small survey amongst other adults to see if the conjecture could gain further support. The scope of the survey was limited and the sample of adults chosen does not represent a cross section of society. This is because the purpose of the survey was not to provide evidence for the proof of my conjecture, but simply to see if there might be any support for it.

The sample was made up of 40 adults with a maximum age of 75, and with a cross section of ages. However the sample was biased in favour of those with a sound mathematical understanding in terms of examination results. The majority of participants had at least the equivalent of a GCSE pass grade at A, B or C. Several had an A level in mathematics and a few had higher qualifications. This of course is not typical of the population as a whole.

The questionnaire (Appendix 1) started by asking the participants to share their attitude to mathematics as a subject at school. Here I found the answers surprising. Considering the high level of examination success most had achieved, reactions to the subject were generally, although not always, negative, with confidence low. The following response was not atypical. "I couldn't understand maths at school. In the end I found it easier just to learn certain facts without full comprehension." This was from someone with a maths 0 level. Here was another common reaction. "Demoralising and baffling." And this was from someone with A level maths!

The questionnaire asked the participants 'to relate in pictures, diagrams or words what you see or think about when you have to quantify the following:-' Next followed a variety of measurements with two examples given. I also discussed the form orally with each person before they filled it in. They were encouraged to leave blank any section where no images or thoughts came immediately to mind. As I talked to the participants I was acutely aware of the point made by John Chatley, 1988,

Gaining access to images is both very simple and profoundly difficult. I can, and having become more aware of them, regularly do, take notice of my own images and my use of them. Access to other people's images is more difficult and there is no guarantee of the reliability of the

information gathered for no matter how good I may be at describing or sharing my image you can only form your own image of mine.' (Intro) (my emphasis)

This proviso needed to be kept in mind during the whole process.

The questionnaire covered measures in the following areas:-

Distance

Area

Weight

Capacity

Time

Some findings

For the sake of clarity I have chosen to give the findings under the five separate headings.

Distance

This was the area of investigation that showed the most constant use of imagery.

Many people saw 1 inch as the top of their thumb, others visualised the inch on a ruler or tape measure. One person thought of a nail, another a snail. One hundred yards was frequently visualised as a sprint, a running

track or in terms of a race; a couple of people thought of it by reference to their gardens and one person saw a swimming pool. The two mileages were thought of in terms of journeys. So we get for 100 miles - 'Southwold to Hatfield' and - 'Distance on a map between Carmarthen to Bristol'. And for 1000 miles - 'Tunbridge Wells to the Costa Blanca' or - 'Land's End to John O'Groats'. However 1000 miles was found to be difficult to image by a number of participants and they left it blank or said they had no reference for it.

Area

Area measures were strongly visualised, except where people found they had little idea of the measure at all as was true for many people when considering 100 square miles. The acre brought a large response of 'a field'; other references made were to gardens, farming land and football pitches. A square yard brought more varied images: card table, carpets, desk tops, material, paving slabs and standing space are examples. Many people also responded to this measure by drawing a square labelled one yard by one yard.

Weight

There were only a few blank responses given under this category. The imagery relied heavily on items used in the kitchen. For example one ounce became a handful of flour, a lump of butter, a tablespoon of sugar or pepper, an egg, a small bag of sweets, and even the metal weight used on old fashioned scales. One pound was for the most part imaged in terms of bags of flour or sugar, jars of jam or honey or in terms of fruit and vegetables. There were individual images like trout and cheese and the metal weight featured again. Ten stone was with few exceptions seen in relation to people's weight, possibly their own or that of someone they knew. One person saw it in terms of potatoes and another a large rock. A ton measure brought some of the following responses: an elephant, a lorry, cannon balls, a pile of gravel and a car. But it was coal which featured the most strongly, coming up again and again.

Capacity

The images raised by the capacity measures showed the least variation, in particular the pint measure. Almost without exception this gave rise to 'milk' - 'a bottle of milk' or 'beer' and its variants. The litre measure conjured up a greater variety of responses, including wine bottles, fruit juice cartons, lemonade, oil, boxed milk and measuring jugs. Several

people imaged a litre with reference to a pint e.g. $1\frac{3}{4}$ pints. For the majority of participants the gallon was visualised in terms of petrol but some people had pictures of containers such as a watering can, water tank, flagon or bucket and one person saw it in terms of 40 miles of travel.

Time

This section brought the greatest variety of responses which is perhaps not surprising since it also contained the largest number of questions. For some people images sprung readily to mind. Others left several blanks on their sheets. Possibly too, some responses indicate associations, rather than images which could be used in tackling measurement tasks. The following examples show the variation in response:- **a century** - the passing of a 100 years, a good cricket innings of 100 runs, the movement in time from one important event to another such as Henry VII's ascension to the throne to Queen Elizabeth I's reign, **1 year** - the cycle of the four seasons, Spring, Summer, Autumn, Winter, 365 days, the full circle of months from one September till the next, the time taken from Christmas Day until the following one, **8a.m.** - breakfast time, work, the news, a cock crowing, insulin and a clock face, **midnight** - full moon, striking clock, lovely sleep, very tired, darkness and bedtime. A **millennium** was a difficult concept for many people and was frequently left blank. Where there were responses

these sometimes showed a blurred understanding such as 'time beyond knowing', 'title of a sci-fi novel' and 'too large to imagine'.

What does this sample show?

As I have already indicated this questionnaire is limited in its use. However what does emerge from the responses is that many of the adults involved in the survey do carry with them a body of imagery when thinking about measurement and measure terms. In many cases the images are vivid and very real. There are of course large differences between individual people. Some are very aware of the imagery that they use whilst others found that fewer images sprung to their minds. What seems true of this particular survey is that all those taking part do at some time or other invoke images in their thinking about measurement; and many of them do use images to a great extent.

Chapter Three

Vivid Images in Mathematics



Chapter 3

Vivid Images in Mathematics

One of the first people to point out the importance of creating and using images in mathematical work was Mary Boole.

Mary Boole lived from 1832 - 1916 and her ideas were in large part inspired by the work of her husband. George Boole, who died in 1864, was a mathematician who held the Chair of Mathematics at a college in Cork. In 1854 he published a work called *An Investigation of the Laws of Thought* and Mary Boole herself became intensely interested in how people actually thought. In addition to her husband's ideas she was also influenced by James Hinton's views. For a short time Mary worked as his secretary. Hinton, whose profession was that of an ear surgeon, wrote on ethics and he also wrote a book about imaging in four dimensions, using colour. Mary Boole had many interests and wrote on several themes but it is her ideas on the importance of using vivid images in mathematics which are of particular interest.

Let us take as an example from one of her lessons, *Lesson on zero*.

Shut eyes, etc. Make a mind-picture:- Me lifting the chalk to the black-board. I make one stroke and then put my hand down. I do this action three times; how many strokes will be on the board? If, instead of making one stroke on the board, I made two and put my hand down; how many strokes would be on the board when I had done the action once? Twice? Three times? Four times? Before I had done it all

Open eyes, sit up. Shut eyes, etc. Make a mind-picture:- A clean black-board, me holding the chalk and then putting it down, without touching the board; what would be on the board? Nothing. Now make another picture:- Me making a stroke. Now I rub the stroke out. What is on the board that you now see in your mind? Nothing. So if I do nothing or if I make a stroke and rub it out, the result is the same as far as the board is concerned. Were the two ways of getting it the same? No.

That was a mind-picture black-board. (p 30)

Here she is emphasising the pictures which the children are to see in their minds. Indeed she actively helps them to build up these pictures, they are an integral part of her teaching method. See again from her *Lesson on curves*:

Try to fancy that this black-board is a field. It has a brick wall all round it, with no opening except at A, where there is a gate.

At Z there is a rabbit hole. A rabbit came out into the field at Z and wandered about till he came to here (write the figure 1), where he found something he liked to eat. After a little while, a dog came in at the gate A; the rabbit caught sight of him, and directly afterwards the dog caught sight

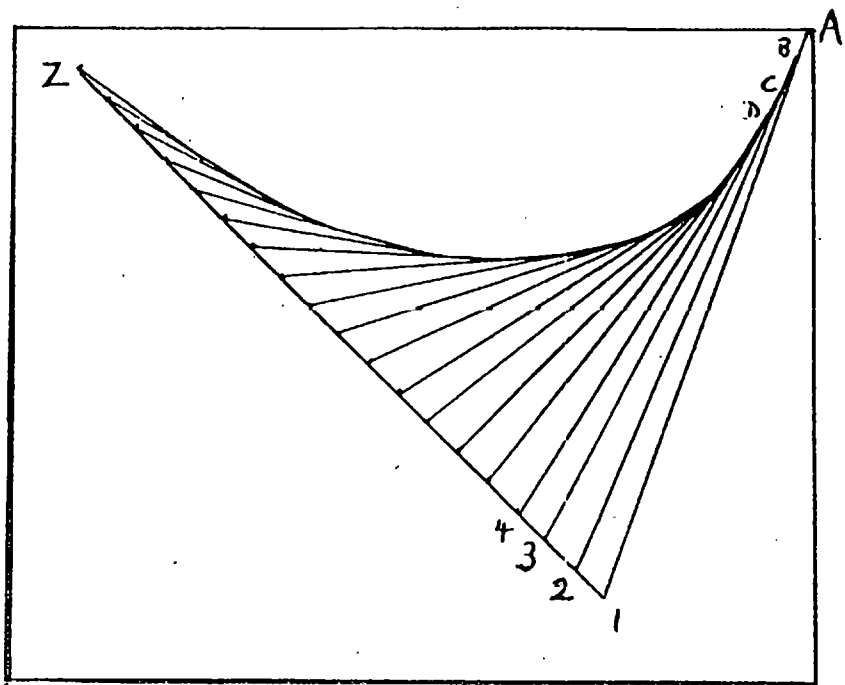
of the rabbit. Now let us try if we can to make out what happened. First we must try to think what the animals would each like to have happen; the rabbit saw the dog first; what do you think he wished? To get away. Perhaps his first idea is to run straight away from the dog. But he can't; the wall prevents him. What will he do next?

Now tell me, what do all these lines represent? The line 1 to Z represents the path which the rabbit would like to go along in one jump and does take in eighteen jumps. The lines A1, B2, C3, and all the other straight lines, each represent a line that the dog at some moment wished to jump along; he jumped along a bit of one and then changed his mind and jumped a bit of the next, and so on. We drew all those straight lines; you saw me draw them by the ruler, did you not?

But here is a curved line A to Z. Who drew that? I drew no line except straight ones by the ruler. Look at it well. Make sure that you see it and all the lines on the black-board.

Now sit slack, shut eyes, and think what the curved line is and how it came. Open eyes and sit up. What is the curved line? The path which the dog really ran, when at each step he meant only to go down some straight line. (ps 32-33)

(See Figure 1 on following page.)



(This is my own diagram)

As before she stresses the importance of the children actually closing their eyes and creating a picture within their minds. In this case the children are encouraged to see the formation of the curve as the path run by the dog. Mary Boole felt that these pictures helped the learning process and the understanding of mathematical ideas. She believed that it was essential to satisfactory learning for a child to see mental pictures linked to the subject matter; and she felt that as a teacher it was her job to help the child to build up the images. In her own words:-

The pauses of sitting with slack muscles and taking slow quiet breaths, to make mind-pictures, . . . are an integral part of the Logic lesson itself

(p38)

and:-

Between the time when a child handles an actual cube, cuts sections, etc., and the time when he comes, among his ordinary geometrical exercises, to problems requiring him to draw the elevation of a cube cut in some particular way, there is a period when he finds it useful, and very delightful, to go through a set of processes in imagination and to express them in his own words. (p 38)(my emphasis)

Mary Boole was not alone in her views. We find support for her ideas in Galtons's book *Enquiries into the Human Faculty* published in 1883. In his introduction Galton outlines his general purpose in writing the book.

My general object has been to take note of the varied hereditary faculties of different men, and of the great differences in different families and races, to learn how far history may have shown the practicability of supplanting inefficient human stock by better strains, and to consider whether it might not be our duty to do so by such efforts as may be reasonable, . . . (p 1)

We might wonder how the subject matter might be of interest to us here! We find the answer in his chapter on Mental Imagery. Galton is interested in the faculty of visualising and sought to obtain statistical data to demonstrate use of this faculty. He came to various conclusions from the analysis of his data, such as:-

. . . that scientific men, as a class have feeble powers of visual representation. (p64)

He accounts for this in the following way.

... an over-ready perception of sharp mental pictures is antagonistic to the acquirement of habits of highly-generalised and abstract thought, especially when the steps of reasoning are carried on by words as symbols, and that if the faculty of seeing the pictures was ever possessed by men who think hard, it is very apt to be lost by disuse. (p60-61)

Galton goes on to discuss the way in which visualising relates to artistic work and claims that 'there is abundant evidence that the visualising faculty admits of being developed by education.' He quotes from the work of M. Lecoq in Paris. We may wish to overlook Galton's findings about 'scientific men' but the point of interest lies in the idea that visualising can be developed by education. This was exactly what Mary Boole was trying to do. Of interest too is his work on Number-Forms. He gives diagrammatic examples to show the way in which different people see number forms, or what in Primary Education today is called the number line. It is a picture in a person's mind in which each number has a certain place. The number line is not only used in Primary Schools but can be a useful general image for negative numbers, rational numbers, decimal numbers and others. In the sample of evidence he uses, Galton discovers that these forms had their origin in early childhood.

It is beyond dispute that these forms originate at an early age; they are subsequently often developed in boyhood and youth so as to include

the higher numbers, and, among mathematical students, the negative values.

Nearly all of my correspondents speak with confidence of their Forms having been in existence as far back as they recollect. One states that he knows he possessed it at the age of four; another that he learnt his multiplication table **by the aid of the elaborate mental diagram he still uses.** (p 86) (my emphasis)

His book actually contains diagrams showing some of the different ways in which some of those he questioned saw numbers, and they are both informative and entertaining.

More recent work exploring the use of mental images in mathematics has also focussed on the use of number and number lines. Plunkett, 1979, argues for the importance of helping to provide children with images in number work:-

My main argument is that for number work children should be provided with visual images that they can internalise and have permanently available, and which can aid their understanding of numbers and their relations.

Commenting on another article he wrote in 1983 he shows that he feels today's teachers do not on the whole work with imagery when teaching children.

The amazing fact remains that, by and large, we do not work with learners on their imagery. No, I am not proposing the imposition of a set of images on reluctant learners. What I am eager to point out is that in doing mathematics imagery seems to be pretty important, and that this fact goes largely unacknowledged in mathematics teaching. Students may well be offered some sort of image but they are rarely invited to work on this. (from Comment 3)

Caleb Gattegno (1911-1988) has contributed a great deal of thought to the area of imagery. Like Mary Boole, Gattegno was a teacher and his ideas were practically applied in the teaching arena. Gattegno was interested in the importance of imagery within the range of geometrical work. Writing in 1965 he quotes from the direct experience of teaching.

In a number of experiments I have asked my classes to consider with their eyes shut some situation in their mind which I generated by instructing them to produce some images and act mentally upon them . . .

The first obvious conclusion reached by all was that the stuff of geometry was the mental stuff called images and the content of the image was directly reachable, changeable at will; a source of relationships which one's awareness could formulate as one or more theorems. (p38)

In this paper he draws a distinction between geometry, as being **an awareness of imagery**, and algebra which he calls an awareness of dynamics. Mathematics as an overall umbrella is seen as an awareness of relationships. Gattegno makes a plea for the greater usage of imagery in

the teaching of young children. It is, he writes 'one of the attributes of the mind that every one brings to school with himself'. (p 38)

Perhaps here would be a suitable place to examine exactly what we *mean* by a 'vivid image'.

Vivid

The *Concise Oxford Dictionary*, gives the following meanings to the word:-

[of light or colour] - bright, intense, glaring

[of person] - full of life

[of mental faculty or impression] - clear, vigorous, strongly marked

In the present context I am concerned with mental faculties and impressions and for such the dictionary would give the meaning to be 'clear, vigorous or strongly marked'.

Does the following example help to give greater clarification to this meaning?

Take the contrasting patterning on two pieces of material. On one is a mass of bright colour, but so arranged that the threads take on a definite structure and a clear motif can be seen. This is a vivid pattern. On the other there is again a combination of colour, but this time in pastel shades and the colours overlap and merge in no organised way. The effect is

mottled and subdued, the pattern cannot be called vivid, indeed no definite pattern is seen at all.

There is a difficulty in finding too concise a definition since images are personal and what one person would describe as 'vivid' might not match up to that of another person. Vividness is a property of the experience of the individual. However for my present purpose I will accept that vivid means 'clear, vigorous or strongly marked'.

Image

Using the *Concise Oxford Dictionary* we find:-

- (i) artificial imitation of the external form of an object
- (ii) optical counterpart produced by rays of light reflected from mirror, refracted through lens
- (iii) form, semblance, counterpart
- (iv) type
- (v) simile, metaphor
- (vi) idea, conception

There are a whole range of meanings here. (vi) would seem to be the one nearest to what I am hoping to express. Yet I feel the definition lacks clarity; 'idea' and 'conception' seem too vague in their signification.

What do we mean by an image?

We need to build upon the definition given in terms of 'idea' and 'conception'. Of help here may be the schools of thought categorised by the two labels 'pictorialist' and 'descriptionalist'. Block, 1981, edited a book entitled *Imagery* contrasting the two views. During his introduction to the book he explains the differing views in the following ways. Of the pictorialists Block says they:-

... agree we don't literally have pictures in our brains, but they insist nevertheless that our mental images represent in roughly the way that pictures represent (p 2)

The descriptionalists are not quite so prescriptive in their formulation of an image. None the less it is still fairly specific.

We should think of mental images as representing in the manner of some non-imagistic representations - namely, in the manner of language rather than pictures. (ps 2, 3)

There is no need to come down in favour of either view. People may not all image in precisely in the same way. As John Mason, 1986, writes:-

Although the words 'imagery' and 'seeing' are based on pictures, I find it useful to encompass a wide variety of inner experiences by the word 'imagery'. (p 2) (my emphasis)

and

... there are widely differing descriptions of the mental screen (p 5)

This view was born out by Presmeg, 1986, in her study of visually mediated processes. She found that the images used by the participants in her fieldwork fell into five different categories. These included pictorial images, dynamic images and memory images of formulae.

For the purposes of my study I would like to move away from the Oxford Dictionary's rather vague definition (vi). However I would not want to come down in favour of a narrow interpretation. Instead I would prefer those used by Mason and Presmeg, in which it is accepted that individual people may form their images in different ways.

Having defined our terms we can join in with the debate and investigation into the use of vivid images which started well over one hundred years ago. Although this debate has been going for so long, the importance of mental images within mathematical processes of thought still remains unclear, and in particular how this may affect the teaching process. As Gattegno, 1967, writes

. . . we can view the process of teaching, even teaching the most elementary mathematics, as a set of opportunities which can enable us to catch ourselves functioning in a variety of ways that **have yet to be collected, catalogued and described.** (my emphasis) (p 28)

Here he acknowledges that there is a great deal more to be explored in how children develop mathematically.

But what of the role of imagery in measurement and measure related mathematics?

Chapter Four

Estimation in the School Curriculum



Chapter 4

Estimation in the School Curriculum

School mathematics topics today usually derive their place from the National Curriculum, which is used by teachers to inform their planning. Because of its pivotal influence on the content of classroom teaching a close look at measurement within the National Curriculum can help us to gain an idea of the measurement tasks generally covered by primary aged children.

Measurement as a topic area was originally dealt with in an attainment target of its own, attainment target 8. Then under the 1991 changes, when the number of attainment targets although not the overall content were much reduced, it was found in targets 2 and 4. AT2 was entitled 'Number', and measurement was found in strands ii and iii, headed 'Estimation and approximation' and 'Measures' respectively. AT4 was entitled 'Shape and Space' and we find 'Measures' under strand iv.

As the statements of attainment unfolded it became immediately clear that children were expected to be proficient in measurement and measurement

estimation when they were still at an elementary level of schooling. For example at level 2 children were expected to 'recognise the need for standard units of measurement' (p 6). At level 3 they should be able to 'Make estimates based on familiar units of measurement, checking results' (p 7) and 'interpret a range of numbers in the context of measurement or money'. (p 7) At level 4 they were expected to 'Make sensible estimates of a range of measures in relation to everyday objects' (p 8).

Examples were given in the National Curriculum document to show the kinds of estimation in which the children were expected to be proficient by level 4. 'Estimate the length of a car, the capacity of a teacup, the weight of a school bag.' (p 8) and 'Estimate the time taken to complete a task.' (p 8).

Of course the National Curriculum underwent further streamlining during 1995 with the new structure brought in by the Dearing Report. Once again the Mathematics Attainment Targets were reduced in number and once again Measures found a new place in the scheme of things. This time all the Measures work is combined with the section on Shape and Space and the two sections are amalgamated into AT3. Although the total layout of the National Curriculum and the Programmes of Study are much

abbreviated with far fewer examples of work being shown, if we read the new Programme of Study it is clear that the children are supposed to be given much the same experiences as before and much the same expectations of their attainment are laid down. So at Key Stage 1 we find:

compare objects and events using appropriate language, . . . ; begin to use a wider range of standard units, including standard units of time, choosing units appropriate to a situation; estimate with these units (p 25)

And at Key Stage 2:

choose appropriate standard units of length, mass, capacity and time, and make sensible estimates with them in everyday situations; extend their understanding of the relationship between units; . . . choose and use appropriate measuring instruments (p 29)

This is virtually the same wording as the 1991 document where children needed to 'Make sensible estimates of a range of measures in relation to everyday objects.' (p 8)

Within my own study I have chosen to focus on measurement estimation within measures work. As John Clayton, 1992, says 'Estimation is a vital part of the measurement process'. I feel that the National Curriculum references to estimation do tie in with my chosen focus, in particular the National Curriculum requirement for children to 'Make sensible estimates of a range of measures in relation to everyday objects' although, as I hope

will become apparent from my work, this requirement makes a simple statement of a complex and many faceted skill. The National Curriculum statements also imply a bland uniformity between estimation in the measures of length, mass, capacity and time, where distinctions may exist. However, despite these points, estimation does find a significant place within the National Curriculum and so we would expect to see this reflected in the teaching of measurement and measurement skills within the classroom and illustrated by modern teaching schemes. In contrast then comes my own personal classroom experience which indicates that children often experience difficulty in estimating. I will look into this claim later.

What is needed first is a workable definition of estimation.

Bright, 1976, saw estimating as:-

the process of arriving at a measurement or measure without the aid of measuring tools. (p 89)

and he goes on to add:-

Estimating is guessing, but the guessing must be educated. (p 89)

Bright's definition is in relation to measurement only. John Clayton, 1988, has a similar definition.

The skill of making an educated guess as to the value of a distance, cost, size number etc: or arithmetic calculation. An estimate then will be defined as that 'educated guess'. (p 9)

Because Clayton's definition refers to more than just measurement estimation it has a wider applicability than Bright's. In reaching his definition he amalgamated definitions by Hall, 1984, and the Collins dictionary. Hall's definition emphasized the mental aspect of estimation:

Estimation is the mental skill of making an educated guess. (p516)

I feel that this is a valid emphasis. The mental aspect of estimation is important. I would want to emphasise this point in my own definition.

Clayton, 1992, feels that his own

definitions are consistent with the use of these terms in the National Curriculum <DES 1991b>. (p 11)

This may be useful, even essential, where a research question demands such consistency; but otherwise I would see no inherent need for definitions of mathematical terms to fit the use of those terms in the National Curriculum.

His definition contrasts with that used by Sikes, 1990,

To form an approximate judgement or opinion regarding the value, amount, size, weight etc., of; to calculate approximately. (p 12)

I do not feel that Sikes's definition makes clear that estimation is a mental skill. Need 'calculate approximately' necessarily be mental? Surely some further definition would be necessary to ensure this is the case. It is possible to calculate approximately using paper and pencil. I also feel that Clayton's, Hall's and Bright's definitions all lay more emphasis than does Sikes's on estimation being a skill based on previous learning. This emerges strongly from their idea of an 'educated guess'. Possibly Sikes assumes this when he writes 'form an approximate judgement or opinion', but this assumption is not clearly stated.

In my own work then, I am understanding estimation to be:- **The mental skill of making an educated guess as to the value of a distance, cost, size, number etc.**

With this definition in mind I have moved on to examine what current writing and educational practice have to say about the possible link between estimation and imagery.

IMAGERY AND ESTIMATION

'We have paid too little attention to the role of images
in mathematical understanding.' (Higginson, 1982)

Measurements frequently summon vivid
images. Could this have consequences for
the teaching of measurement estimation
skills to junior aged children?



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Volume II of III

Chapter Five

Does Imagery have a Role in the
Estimation Process?



Chapter 5

Does Imagery have a Role in the Estimation Process ?

Imagery in the estimation process seems to be of paramount importance; so writes Margaret Brenchley, 1986, while dealing with young children's estimation skills in her study *The Scales of my mind* (p 70). In this study, which involved 12 children aged between 5 and 9 years old, she found that the children used many references to images.

The children's conversations were dominated by reference to it. Only Alex (5.10) and Amanda (6.0) failed to describe any images.

Bright, 1976, also makes constant reference to the importance of mental manipulation in estimation. Mental manipulation here includes the use of mental pictures:-

Presumably, students mentally compare the given unit with the named object and determine the appropriate measure. (p 91)

and later,

... Such activities help students picture mentally the size of a given unit of measure. Presumably they picture the measurement in terms of the repetitions of the unit and compare this mental picture with objects ... around them. (p 92)

and

when the unit is absent, students must mentally picture the unit before they can begin the process of making an estimate. (p 92)

If these researchers are correct then surely we would expect to see some reference to image building within the traditional teaching of estimation skills, since estimation itself is still seen to be of paramount importance in the view of primary school educators. Is this the case? A look in detail at two traditional teaching schemes should give some clues.

Ginn Mathematics was first published in 1983. Its second edition came out in 1990, completely revised to take account of the National Curriculum, and it is a scheme much favoured by schools in recent years. In the teachers' resource book the scheme provides its own succinct introduction:-

The new edition offers a comprehensive, up-to-date programme with a structure and framework of teacher support that ensures a full coverage of content and method in accord with the National Curriculum.

Ginn Mathematics provides for individual, group and class work and for a variety of methods of organizing the classroom activities. Objectives are clearly stated in a carefully mapped out progression. The teaching of mathematical skills is linked to language development and the growth of concepts. These are all presented in an integrated mathematical framework, but advice is also given to ensure the important

inter-connection with other subjects. As mathematical understanding increases, and skills develop, it becomes easier both to use maths purposefully in other curricular areas, and for pupils to recognize the mathematics which exists in other areas of study. (p 3)

This should augur well for structured work on estimation since the National Curriculum, as we have seen before, specifies attainment in this area. In Chapter 4 I looked at the statements of attainment in the National Curriculum 1991 edition. It is appropriate here to consider the programmes of study as well. The programmes of study are considered to be of extreme importance to the teacher and presumably therefore also to the creators of Ginn Mathematics.

As we would expect there is extensive reference to estimation in the NC programmes of study. The term is found not only with reference to measurement work but also in work relating to number and to handling data. We find:-

making a sensible estimate of a number of objects up to 10 (p 5)

making estimates based on familiar units (p 6)

estimating and approximating to check the validity of addition and subtraction calculations (p 7)

making sensible estimates of a range of measures in relation to everyday objects (p 7)

using estimation and approximation to check that answers to multiplication and division problems involving whole numbers are of the right order (p 8)
giving and justifying subjective estimates of probabilities (p 19)

Claiming therefore 'full coverage of content and method in accord with the National Curriculum' Ginn Mathematics should leave ample space for estimation work.

How is estimation work tackled in the pupil text and workbooks and in the book intended for group teaching called the *Big Book*? I have examined the measures work at level 4 of Ginn Mathematics, which is generally tackled by children aged from 8 to 10. The scheme is arranged in stages up to a total of 67 at level 4.

Stage 56 deals with measurement in centimetres. In the teachers' resource book the objectives for the stage run as follows:-

To enable the children to:

measure lengths to the nearest centimetre;

draw lines of given lengths in centimetres;

estimate lengths to the nearest centimetre. (p 118) (my emphasis)

However on the pages of Ginn's text there is no reference at all to any kind of estimation work. Instead the children are simply asked to measure the lengths of some objects drawn on the page and then asked to draw some

lines of certain lengths. They are also asked to measure parts of their bodies. There is also no reference in the teachers' notes to any kind of estimation work. It is obviously assumed that estimation skills will arise through specific measurement.

We find the same thing in stage 57 where measurement in metres and kilometres is dealt with. There is some work on approximation since measurements to the nearest metre are allowed, but this is not estimation. However in this case some reference to actual estimation work is made in the teachers' notes.

Let the children see and handle some metre sticks. Ask the children to estimate the measurements of several objects in the classroom. Keep a record of these estimates and then let the children measure the objects with a metre stick. (p 120)

There is also one significant sentence found a little later under a section called 'Using the pages'.

. . . explain the metre and kilometre again and ask the children what kinds of things are measured with each. Tell them it is always useful to estimate the measurement of distances first. **If they imagine a finger width as being about 1 centimetre and a 'giant stride' as being approximately 1 metre this will help them with estimation.** (p 120) (my emphasis)

This is an extremely important sentence. Here is an attempt to build up some simple images in the children's minds **to help them with estimation.** Unfortunately this instruction is found buried amongst so many other directions and ideas that it is a moot point how many teachers actually notice it and, having noticed it, actually use it. It is also a recognised fact that in some cases junior children just follow through textbooks at their own speed and where this happens then this instruction would be totally lost.

When looking at weight we find a similar situation. The children's pages make no reference to estimation work but some reference is made in the teachers' notes. For example regarding the 1 kg weight, the children are to be encouraged to 'hold it and feel its weight, then use the experience for estimation'.

Work on area does not even attempt to make estimations, with no reference made even in the teachers' notes; so we pass on to capacity. Here we find the children are actually asked to estimate in the written instructions of the text book. This is found at the bottom of the page under 'Project 2'.

- (a) Collect some containers and estimate how many litres they will hold.

(b) Check your estimates by filling each container, using a one litre jug.

(p 125)

Estimation is encouraged in the teachers' notes too but there is no specific input in building up imagery to help with this as we saw in the notes on length.

It is in the section in Ginn devoted to time, stage 39, that we find the closest efforts to build up imagery in children. Under a lesson plan for using the big class book, the *Big Book*, is the following:-

Discuss the passage of one minute. Ask the children what jobs they think they can complete in exactly one minute. Ask one half of the class to stand up, eyes closed, while the other half looks at the clock. On a signal from a child, the children standing up each have to estimate the passage of one minute and then sit down. The children keeping time should take note of when each child sits down and discuss the results.' (p 88)

The relevant point here is found when the children are asked to think of the jobs they can do in one minute. To establish meaningful imagery the children need to identify a job **which actually does take one minute**. If this is not done then the child will not have built up any valid imagery. For example let us suppose that Ted thinks that putting on his lace up school shoes in the morning takes one minute. When it is time for the class exercise and it is his turn to stand up he closes his eyes and images this

process, sitting down when it has been completed in his mind. If this job in fact only took forty seconds then simply discussing the error within the class will not greatly help Ted to build up a useful image of one whole minute. He needs repetitions of this task until he finds a job which *does* take approximately one minute. Extensive work may be necessary for him to achieve this. However the instructions to the teacher are not clear on this point.

What have we discovered about Ginn Mathematics? Firstly on the positive side there are a few references to building up images to help in estimation work, most specifically to do with length and the estimation of the centimetre and metre, and of time and the estimation of one minute. Unfortunately this is but a drop in the ocean. The text books used by the children have surprisingly little estimation work at all and what they do have doesn't specifically attempt to build up useful images. In the teachers' notes there are some such references but only in certain fields of measurement, they are not stressed and they can be misleading.

Heinemann Mathematics was first published in 1992. It was created to take the place of Scottish Primary Mathematics and was written by the SPM Group. There follow below some of the aims of the new scheme.

The course is based on the belief that mathematics is best learned through practical activities, discussion and teaching by the teacher. The use of materials, diagrams, and pictures to help pupils acquire concepts and understand techniques is encouraged throughout the Heinemann Mathematics course. . . .

Mathematics is presented in context wherever possible so that it can be seen to relate to the world outside the classroom **and the world of the child's imagination**. Such contexts are more likely to stimulate an interest in mathematics and encourage positive attitudes. . . .

The approach adopted in Heinemann Mathematics is in tune with the guidelines provided by the National Curriculum (England and Wales), . . . The course has been designed to provide teachers with resources and a structure to meet the requirements of each curriculum. Assessment material linked to attainment targets is included.' (preface), (my emphasis)

The scheme promises full National Curriculum coverage and therefore, as with Ginn, there is the strong hope that there will be ample coverage of estimation and the development of estimation skills. The part I have emphasised may, or may not, indicate that there will be some reference to the building of children's images within their work. It could refer simply to the context in which the subject matter is placed and not to internal imaging.

I have chosen to examine Heinemann at level 3, usually tackled by children aged between 7 and 9. For children working at this level there is a workbook to be completed on 'Measure, Shape and Handling Data' and an accompanying textbook. A book entitled 'Teacher's Notes' runs parallel to the children's material.

Looking first at distance work on page 8 it is interesting to see that before any work on estimation is attempted the children are encouraged to build up some idea of a metre by finding two objects which are about 1 metre long. The Teacher's Notes read 'Experience of measuring objects fairly close to 1 metre long is useful in building up their concept of a metre'. (p 149) Unfortunately the children's books simply give the children two chances to find objects of roughly one metre and no practice in imaging. It does seem as though the writers of the scheme have given some thought to the importance of creating images but this is not carried out in the actual work given to the children. On the following page of the Notes we find:

Introduce the idea of estimating each length before measuring. The children may guess wildly at first. This should improve if children are encouraged to visualise how many metre lengths will fit along the length to be estimated. (p 150) (my emphasis)

This will of course depend upon whether they have formed an accurate image of a metre length.

The section on weight goes directly into estimation without any attempt to help the children build up a meaningful image of a kilogram weight. The Teacher's Notes are more helpful but they only offer the teacher one exercise.

Introduce the word 'kilogram' by discussing buying potatoes, sugar, etc.

Pass round a packet of sugar and a commercial 1kg weight for each child to hold. (p 162)

This is surely a good start, but only a start.

There is no work on estimation in the section on area in the children's books. This is the same in the Teacher's Notes. The work covered is dealing with measuring area by counting squares and the idea of conservation of area.

Capacity work covers estimation and before the children tackle the work they are at least encouraged to find containers which hold about 1 litre of liquid. Again the success of this operation will depend on the class collection of containers and the individual teacher's use that she makes of this part of the page. There is no reference in any text to visualising a litre measure or to exercises helping to build up a useful image for the child.

Like the Ginn Scheme it seems that it is the work on Time that really tries to build up an image of a minute in a child by encouraging them to undertake activities that take one minute. There is a section in the Teacher's Notes which is called 'CONCEPT OF A MINUTE Introductory activities'. Here we find:-

1. Sitting silently for one minute.

Indicate the start and finish of the time interval of one minute. Suggest to the children that they silently count 'one, two, three, . . . ' over the period.

This should help them with activities where the duration of a minute has to be estimated.

Although the counting itself will not produce a meaningful image, if the children then work on other activities, using the counting to help them establish what takes one minute, a meaningful image may emerge.

So the new Heinemann Mathematics does make reference to the practice of imaging in *some* of its estimation activities in measurement. However these often need to be searched out carefully from the midst of copious Teacher text, are not structured with enough detail and do not cover all areas of measurement.

The conventional and widely used mathematical schemes which I have examined do provide work which gives primary children the chance to

make estimations when they are dealing with Measures. However very little is built into these tasks to give the children any real help in this work. For the most part they are just expected to get on and make estimations. No suggestions are made as to how to estimate, other than guessing. It seems to be implied that practice makes perfect, (although in some cases very little practice is given). The building of valid images which would enable them to make sense of what they are doing is for the most part ignored. Yet, as I have already noted at the start of this chapter, recent research has already pinpointed the importance of valid image building in estimation.

Let us look at Margaret Brenchley's work again. Affirming the importance that image building has in estimation work she finds that children in the pre-operational stage are not yet able to use their images alone when they make estimations. They need to combine mental images with practical observation, but this stage can be left out by older children:-

If children in this transitional period are unable to manipulate their images or transfer them to "benchmarks", it must mean that when making estimates they should have freedom of movement to actively explore the area to be estimated (i.e. move across a room). Mental images will not be sufficient for the estimation to be carried out. **Then by the time the operational period is reached the child should be able to use past**

experiences and mental images of the experiences to estimate successfully. (ps 68, 69) (my emphasis)

I think that stress should be laid upon the word 'should'. Can children do this when at the operational stage? Have they had sufficient input into the formulation of measurement images for confident estimation to be made?

The importance of this was supported by a study whose findings were published in 1990. This study was undertaken by Forrester, Latham and Shire from the University of Kent. In the discussion which comes at the end of the article we find:-

Consider first, factors important to background skills. In contrast to previous studies, which have either glossed over or ignored the role of imagery, the children's explanations placed imagery central stage in their attempts to formulate an answer. (p 296)

While we attempt to build up mental imagery in children to help them in their work in estimation we will also need to clear away some of the misunderstandings that children themselves have formulated about such work. I have frequently found in my work with junior children of all ages that they associate estimation work with getting right answers.

This was born out by some work on probability which I undertook with a class of 28 junior children in the Summer Term 1993. The children were a mixed group of year 4, year 5 and year 6 and roughly one half of them have some sort of learning difficulty sufficient to take them on to the Kent Special Needs audit. I found that many of the children would rub out their estimates if they proved wrong and change them to coincide with the actual answer. For many in this class the notion of estimation as an 'educated guess' was a foreign concept. An estimation was supposed to be 'right'.

This point is reiterated in an OU publication entitled 'Measuring' ,1980:-

Adults often use rough estimation techniques either as a preliminary to an accurate measurement or to give some idea of the magnitude of the quantity. **The accuracy of this sort of estimation is not important** - the purpose only requires a rough guideline or else it is a preliminary to another more accurate measurement. Many children do not realise the value of rough estimation. (p 43) (my emphasis)

Brenchley, 1986, confirms this view:-

Textbooks all recommend estimation but none suggest its purpose (Liebeck 1985). This leads to children construing the activity as purely a competition for the right answer. (p 72)

So too does Sowder, 1992:-

The need for finding an exact answer is instilled in children from the early grades on. (p 379)

and

Students must see the relevance of learning estimation and must believe they are capable of doing it. They will need to overcome the belief that there is always one right answer and one right procedure for obtaining it.
(p 379)

Children are so often inculcated to attain 'right answers' in mathematics, but in adult life these may not be necessary. As Sikes, 1990, writes:

It is never too early for children to know that mathematics is not always exact and that a single "right answer" is not always attainable. Often a rough approximation is just fine. (p 20)

John Maddox, 1977, made the same pertinent point when he wrote:-

First, adults are much less in need of exact calculations than of being able to make numerical guesses which are accurate. Will five rolls of wallpaper do the job? Roughly how long will it take to drive to the Costa Brava? Is it better to finesse the Queen of Spades or to play for the drop? If the gross national product is roughly £100,000m, is the GNP per head more like £2,000 than £1,000?

The importance of these skills of estimation is widely and grossly underestimated, yet in everyday maths - everyday life - they are the skills that matter. There is a sense in which it is more valuable to know that £1,000 per annum is roughly £20 a week than that it is £19.230769 per week. £20 per week is a number that you can easily use in other calculations, in working out the weekly equivalent of a princely salary, for

example. The other number I have given is useless by comparison.(from Science Diary)

He went on to comment that 'a flair for estimation is one forgotten part of numeracy'.

In daily life children may meet with the following types of problems. In solving each the skill of estimation will help them.

- ♦ How far is it to my friend's house?
- ♦ Will I get there before lunch?
- ♦ The teacher said our margin should be roughly 2 cms, is mine okay?
- ♦ I need an area approximately one square metre to grow my 20 plants.
- ♦ I need $\frac{1}{2}$ litre of milk for two milk shakes. Is there enough in this jug?
- ♦ How much lemonade must I buy from the shop for my six friends?
- ♦ How long will it take to the shops and back - will I be back in time for 'Neighbours'?
- ♦ When must I leave for school in order to get there on time?
- ♦ If that parcel weighs 5 kg will I be able to carry it alone or should I get some help?

Both the National Curriculum and modern educators are insistent on the importance of useful estimation. It would seem to be one of the mathematical skills that we use throughout our lives. Research has shown that the building of valid imagery may be of importance in acquiring these skills. However within traditional maths teaching little is done to aid the child in building up these images. Research needs to be directed at the actual ways in which teachers may help to build up this imagery in their pupils. However even as these different ways are explored we need to be aware of an extremely important point made by Bishop, 1989:-

. . . visualisation and the use of visual imagery is a very personal matter.

Individual pupils need to given time to develop their own images, and there seems little value in teachers expecting identical images to result from such a personal process. We must anticipate, and to some extent therefore encourage the possibility of, diversity in visualisation. (p 11)

Chapter Six

Do Children Experience Difficulty with
Measurement Estimation?



Do children experience difficulty with measurement estimation?

My experience as a classroom teacher would lead me to believe that many junior aged children find accurate estimation work difficult. This is supported by the experience of my colleagues. There is also some support for this view amongst researchers. Clayton, 1988, showed that children make less successful estimates with metric units than with imperial units despite using them in their school work. This had also been born out by the Sutton Test, an aural/visual test given to a mixed ability group of schoolchildren of primary and secondary age in the Borough of Sutton and 'designed to investigate various aspects of estimation'. Sowder, 1992, looks at the results of several studies into estimation competence undertaken in the United States, a study by Crawford and Zylstra (1952), the Corle studies (1960, 1963) and studies by Swan and Jones (1971, 1980). She finds:-

School children and adults are not good estimators. More than 50% of the estimates in the Swan and Jones studies were considered to be poor estimates. As opposed to the 147 students included in the Corle studies,

the Swan and Jones (1980) study included almost 1,300 students, from both rural and urban areas. (p 383)

I therefore set about working with a small group of local children to discover their competency when working on measurement estimation tasks.

The Group

The group consisted of 32 nine and ten year olds in a mixed ability class. Of these 32 children nearly one half were above average ability and within one year would have gained 11+ places at grammar schools. One quarter were of average ability and the final quarter were of below average ability including six registered on the Kent Special Needs register at level 1.

Tasks

As a group test the children were given, orally, a number of everyday or classroom objects which they were asked to estimate in any way that they wanted. Some children chose to use units, either standard or non-standardised, and some used direct comparison. Some again used a combination of the two. The children were also asked to give the reasons for their answers. (See Appendix 2)

These questions include the five different categories of measurement which I used in my adult survey:- distance, area, weight, capacity and time.

Categorising the results.

At first I thought of arranging them in the following ways:-

- 1) Comparison without unit
- 2) Comparison with unit
- 3) Memory with units
- 4) Unrecognisable

I would then have made a further division of the first three categories into reasonable and unreasonable estimation. This would have lead to seven possible outcomes in all.

A second method I considered was to look at the results following a decision tree (see Figure 2). First I asked the question "Do they know?"

The response would take the answer down one of two routes, yes or no.

Taking the 'no' route would end there but the 'yes' answers would proceed along another of two routes: does the child use 'Units' or 'Non-units'.

Taking either of these paths would lead to the next question: was the response reasonable or unreasonable? This categorisation would lead to five different results.

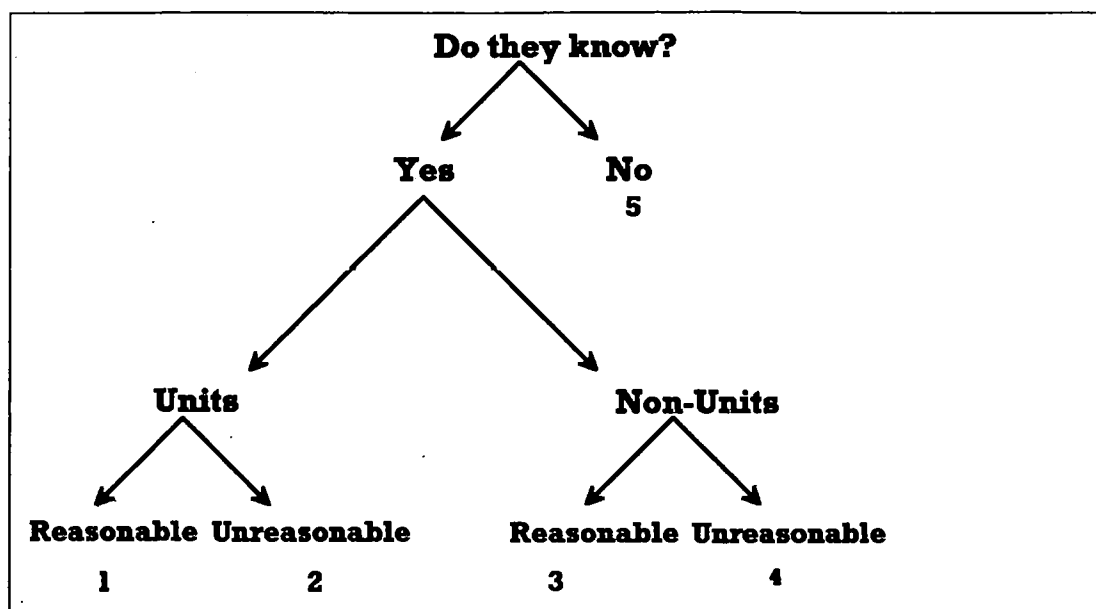


Figure 2

I felt however that neither of these arrangements would do because they didn't differentiate sufficiently between the children's responses. The first way made no allowance for a difference being drawn between standard and non-standard units, something I was keen to establish. The second method didn't differentiate at all between the types of wrong responses and I felt this might be of importance in analysing the results.

The way I finally decided to categorise the children's answers allowed for a larger variety of responses, and in particular a greater variety of unreasonable responses. I used a decision tree as follows (Figure 3).

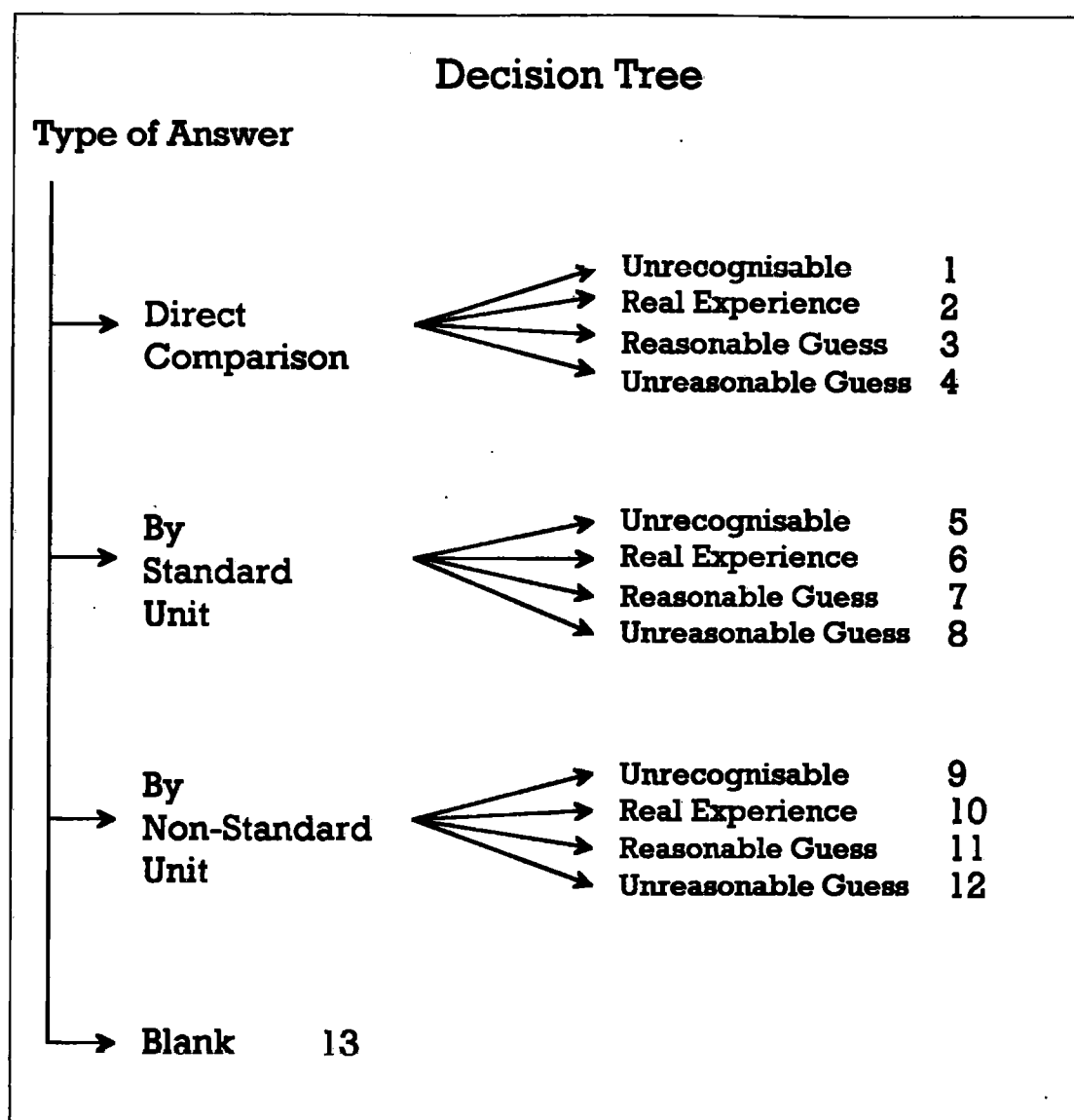


Figure 3

This route branches into a choice of four mutually incompatible paths down one of which each child will have to go: use of direct comparison, use of a standard unit, use of a non-standard unit and finally just a blank left where there should have been an answer. These first three paths then branch *themselves* into four separate paths. These are comprised of the following: unrecognisable, real experience, reasonable guess or

unreasonable guess. Thus the final number of possible responses number thirteen.

Here are the responses listed in a different way and given a code.

- 1) Dn - direct comparison, unrecognisable
- 2) De - direct comparison, real experience
- 3) Dr - direct comparison, reasonable guess
- 4) Du - direct comparison, unreasonable guess
- 5) Sn - standard unit, unrecognisable
- 6) Se - standard unit, real experience
- 7) Sr - standard unit, reasonable guess
- 8) Su - standard unit, unreasonable guess
- 9) Un - non-standard unit, unrecognisable
- 10) Ue - non-standard unit, real experience
- 11) Ur - non-standard unit, reasonable guess
- 12) Uu - non-standard unit, unreasonable guess
- 13) B - blanks

In looking at the children's estimates I felt that at this stage assessing them as reasonable and unreasonable was sufficient for my purposes. What I wanted to establish was some overall sense of whether the children had or

had not made a reasonable estimate of the size, weight etc. of the object under consideration. In addition comparisons between estimates made in different ways, some using standard units, some non-standard units and some direct comparison, would have been difficult to formalise. Later in my thesis when I come to compare estimates more precisely and need a definition of accuracy I formalise this in terms of relative error.

Why did I include the terms 'real experience' and 'unrecognisable'? Their inclusion, which I feel is valid, needs some explanation.

A 'real experience' answer:- I felt that there was an important difference between a reasonable estimation and a reasonable answer given because the child happened to be measuring a few days before and could remember the result. 'Real experience' refers to the latter happening. It was therefore very important to find out why the children made the estimates that they did and so I asked them to give reasons as to why they had made each estimation. They did this as they progressed through the questions. So for example when one child was asked to make her estimation of how tall her mother was she wrote, '1m 50cm because I've measured her'. And how heavy was she herself? '5½ stone because I weighed myself'. Both these answers would be placed in my category of 'real experience'. Asking for

reasons enabled me to make this important distinction between true estimation and simple knowledge. The child is simply using his/her memory correctly and not estimating.

An 'unrecognisable' answer - I included here answers which were undecipherable or simply didn't make sense. In addition I classified as unrecognisable those answers where the child used units which did not correspond to the measure being asked for, like using square centimetres to measure water capacity. Finally I included answers where the child simply wrote descriptive words such as 'big'.

Having decided on the way to analyse my results I wrote the results out in a table. I arranged the questions so that the ones dealing with the same type of measures were next to each other. Along the top of the table were the numbers of the questions in the following order;-

1 7 2 8 3 4 5 10 6 9

1 and 7 are area, 2 and 8 capacity, 3 and 4 length, 5 and 10 weight and 6 and 9 time. Down the left hand side were the names of the children and I used the code to chart their results. The total number of responses totalled 320. (see Table 1)

Table 1

	Questions									
	AREA		CAPACITY		LENGTH		WEIGHT		TIME	
Child	1	7	2	8	3	4	5	10	6	9
1	Du	B	Ur	Ur	Sr	Dr	Se	Sr	Sr	Sr
2	Sr	Su	Ur	Ur	Sr	Uu	Se	Su	Su	Sr
3	Su	Su	Ur	Su	B	Su	Se	Sr	Sr	Su
4	Su	Su	Ur	Se	Se	Ur	Se	Sr	Sr	Su
5	Su	Sr	Dr	Su	Su	Sr	Se	Su	Sr	Sr
6	Dr	Ur	Ur	Ur	Se	Ur	Se	Su	Su	Sr
7	Su	Su	Sr	Su	Se	Sr	Se	Sr	Sr	Sr
8	Ur	Su	Dr	Su	Dr	Uu	Se	Su	Sr	Sr
9	Dr	B	B	Dr	Su	Uu	Sr	Sr	Sr	Sr
10	Du	Du	Uu	Su	Se	Su	Se	B	Se	Su
11	Su	Su	Ur	Su	Sp	Su	Se	Su	Sr	Su
12	Dr	Su	Uu	B	Se	Su	Sn	B	Su	Sr
13	Dr	Su	Sn	Sn	Se	Su	Se	Su	Sr	Sr
14	Dr	Su	Sr	Sr	Sn	Su	Se	Su	Sr	Sr
15	Du	Sn	Un	Su	Su	Su	Se	Su	Su	B
16	Dr	Dr	Sr	Su	Se	Su	Se	Dr	Sr	Sr
17	Su	Sr	Uu	Su	Sn	Su	Se	B	Su	Sr
18	Dr	Dr	Ur	B	B	Su	Se	Sr	Sr	Sr
19	Dr	Du	Dr	Su	Dr	Dr	Dr	Su	Sr	Dr
20	Dr	Du	Dr	Su	Sr	Sr	Se	Su	Sr	Sr
21	B	B	Ur	Sr	Sr	B	Se	Dr	Sr	Sr
22	Su	Su	Ur	Sr	Sr	Sr	Se	Dr	Sr	Sr
23	Dr	Du	Sr	Sr	Dr	Sr	Se	B	B	Sr
24	Dr	B	Du	Sn	B	Sr	Se	Sr	Sr	Su
25	Du	Su	Dr	Dr	Su	Sr	Se	Su	Sr	Su
26	Dr	Su	Su	Sr	Su	Sr	Se	Sr	Su	Sr
27	Dr	Sr	Su	Su	Se	Sr	Se	Su	Sr	Sr
28	Du	Su	Ur	Su	Se	Su	Se	B	Sr	Sr
29	Dr	Dr	Ur	Dr	Se	Su	Se	Sr	Su	Sr
30	Du	B	B	B	B	B	Sn	B	B	Sr
31	Dr	Su	Sr	Sr	Se	Sr	Se	Su	Su	Sr
32	Dr	Du	Ur	B	Dr	Uu	Se	Sr	Sr	Sr

By photocopying the results and then highlighting similar answers I could draw some findings from the table.

The first area I looked at was to see whether the children used direct comparison or units in their estimations.

Only 49 responses were given in direct comparison, less than one fifth. But there were marked differences in the spheres of measurement in which the children chose to use direct comparison. When the children were dealing with area they gave many answers in direct comparison, nearly half. However this contrasts with the other areas of measurement where percentages were much lower. Although the total number of answers given in direct comparison was relatively small it is interesting to see that nearly half of the children used this method when dealing with area.

Perhaps it was particular children who used direct comparison, or was it a random use? The table show that the latter was true. The children answered at most three times in that mode and generally less except for one child who used direct comparison in seven of his answers. The following shows the exact distribution:-

Answers given in Direct Comparison

0 answers - 7 children

1 answer - 12 children

2 answers - 6 children

3 answers - 6 children

7 answers - 1 child

Next I looked at the way in which the children used units when they were estimating. In total 242 answers were given using units. (This number in no way indicates the accuracy of the responses.) This is more than three quarters of all the answers. The interesting point is that of these, 215 were given in standard units and only 27 given in non-standard units.

The spread across the five areas of measurement is not even. Sixteen of the answers given without standard units came from question 2, which asked about the capacity of the class sink. In total almost one third of the answers about capacity were given in non-standard units. This contrasts with the areas of 'Time' and 'Weight' where all the answers were given in standard units.

The next category I examined was the number of answers which were left as a complete blank. This amounted to very few as the questions were given in a relaxed class atmosphere, one by one, and the children had been encouraged to give as many answers as they could. Only 28 blanks were left, just under one tenth of all the answers. They were spread evenly between the five areas of measure, with one question getting rather more than the other nine. This was the question asking the children to estimate the weight of their reading book. There were 7 blanks left, nearly a quarter of all the answers to that particular question. In contrast the other question dealing with weight, which asked the children how heavy they were had no blank answers at all.

Having examined the way in which the children gave their answers I next looked at the reasonableness of their results. I had chosen to categorize the results in four ways: unrecognisable, real experience, reasonable guess and unreasonable guess.

Looking first at the unrecognisable results I found that these were very few in number. Only 9 unrecognisable answers were given. I felt I could sensibly place these unrecognisable answers together with the blank responses, because both indicate confusion with the question. One child

gave eight either blank or unrecognisable answers and he was obviously unhappy with the whole exercise. In all, sixteen of the children gave at least one blank or unrecognisable result. This is half of the whole class.

I next looked at the 'real experience' results. I have already outlined the reasoning behind this category and how these results cannot be considered true estimation, yet it is still interesting to look at this category of results and some clear trends are visible. For a start only four questions were answered with reference to real experience. These were questions 3, 5, 6 and 8. The vast majority of responses came in answer to 3 and 5, question 3 having eleven and question 5 twenty eight 'real experience' answers. Looking at the questions reveals why:-

3) How tall is your Mum?

5) How heavy are you?

These are questions which would be answered in the everyday life of the children. Perhaps these questions would have been better replaced with other ones.

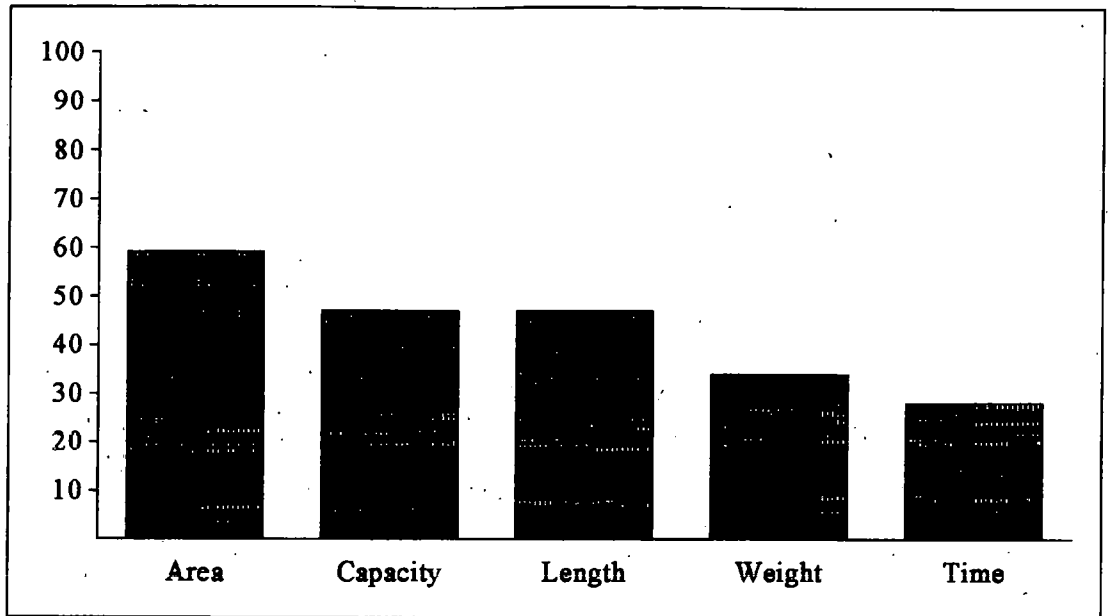
I then examined the unreasonable results. In all there were 101 unreasonable results. If this is added to the 37 answers which were in addition left blank or showed confusion then the total number of

unreasonable answers was 138. This is not far off half of all the answers given and is very high indeed. Especially when it is remembered that the sample comprises a group of above average children.

All the questions had their fair share of unreasonable answers except one of the questions dealing with weight. This was the question which asked the children how heavy they were. Nearly every child had real experience of this and therefore this was reflected in their answers. In contrast the question on the weight of a book produced thirteen unreasonable answers. Remembering that the total of possible answers under each category was sixty four it is interesting to see how many were unreasonable. I have also combined these with the blank (B) and unrecognisable (N) answers and gained a total percentage of these results measured together.

U, B, N RESULTS

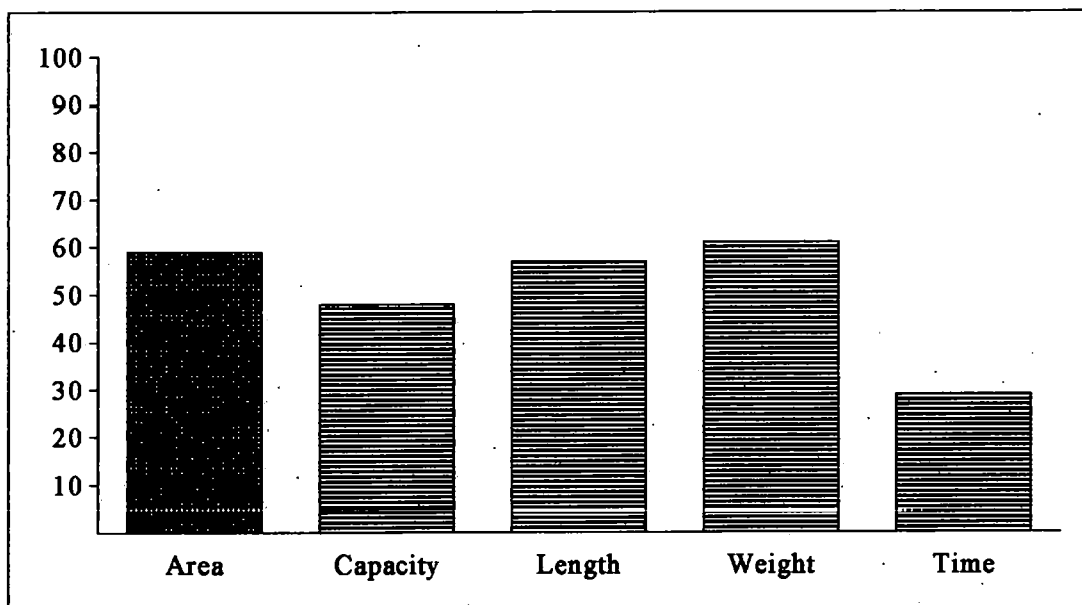
	Unreasonable results	Total (with B and N)	%
Area	31	38	59
Capacity	20	30	47
Length	22	30	47
Weight	13	22	34
Time	15	18	28



This table and its corresponding bar chart show that the most unreasonable results came when the children were asked to estimate area. The estimates of time and weight were more reasonable. However when we remember that so many of the weight estimates were not in fact estimates but 'real experience' on the part of the child it can be seen that these figures do not give a true picture. I have therefore completed another table which shows the total of Unreasonable (U), blank (B) and unrecognisable (N) results next to the number of responses when real experience results are discounted.

U,B,N RESULTS **discounting Reai Experience**

	U,B and N results	No of responses	%
Area	38	64	59
Capacity	30	63	48
Length	30	53	57
Weight	22	36	61
Time	18	63	29
Combined	138	279	49



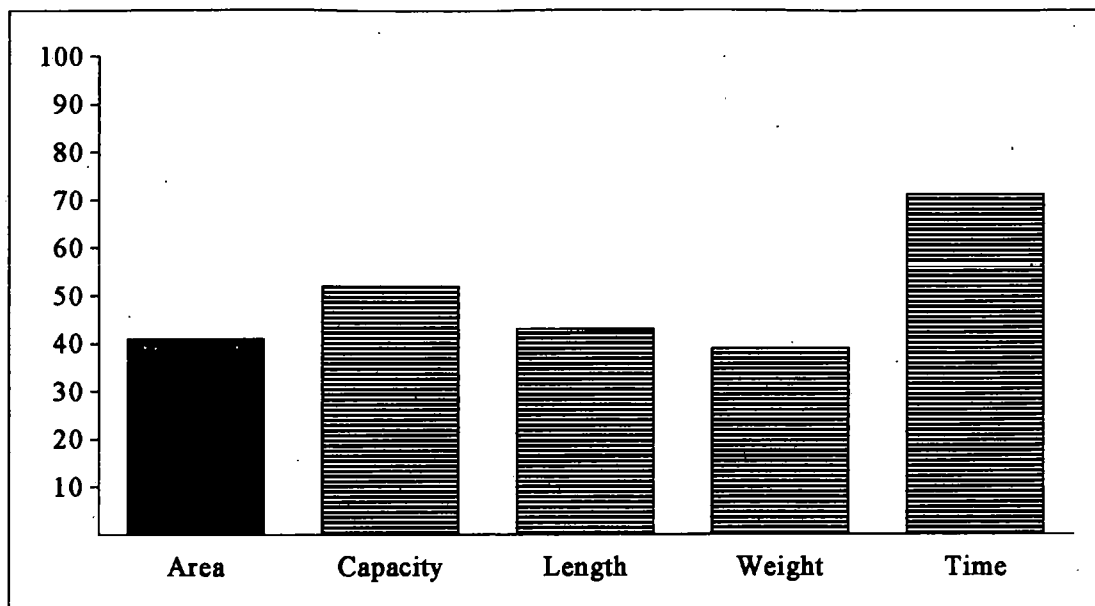
These results are particularly interesting when contrasted with the previous table and chart. Each percentage has increased except for Area which has

stayed the same. Those of Capacity and Time have increased only by one digit, but there is quite a change in the results of Length and Weight, which have both also now got a smaller field of response. Looking at the percentage figures we find that there was unreasonableness overall of nearly half.

Turning now to reasonable answers, we see the obverse results from the ones we have just discovered. Here is the table:-

REASONABLE RESULTS
Discounting Real Experience

	Reasonable results	No of responses	%
Area	26	64	41
Capacity	33	63	52
Length	23	53	43
Weight	14	36	39
Time	45	63	71
Combined	141	279	51



I have analysed the reasonable results still further to see whether they were given mostly in direct comparison, non-standard units or standard units. There were 141 reasonable results in all. Of these the largest number were in standard units, 85 in all. 38 were from direct comparison and 18 using non-standard units. As percentages of the total responses, less real experience results these are shown as :-

Reasonable using direct comparison - 14%

Reasonable using non-standard units - 6%

Reasonable using standard units - 30%

Does this study show us anything?

A body of material has been gathered which has shown the many different ways in which a group of children chose to estimate. It has also given

some indication as to how accurate these particular children were in their particular test.

So what have we learnt? That many of these year 5 children were experiencing some difficulty in fulfilling the NC attainment target 'Make estimates of a range of measures in relation to every day objects'. Looking again at some of the findings:- More than half of all the children gave unreasonable results in estimating area, length and weight and only a third gave reasonable estimation using standard units. This would not appear to be an isolated finding if we bear in mind the findings of Sowder. It would seem that many children do experience difficulty with measurement estimation. In addition there are possibly more general difficulties associated with motivating children to estimate. This may be due in part to a child's own desire to find 'right' answers but also perhaps to a lack of appropriate and relevant estimation tasks within the class setting. Where a child can see a purpose behind the estimation they are being asked to make they are given an incentive in their work.

But if measures summon vivid images, and imagery is essential to competent estimation can children be helped to more accurate estimation by an increased use of imagery? Might my initial conjecture that 'measures

summon vivid images', be further refined to encompass this point. In other words I would like to conjecture that - AN INCREASED USE OF IMAGERY IN ESTIMATION CAN INCREASE ACCURACY.

Chapter Seven

The Estimation Process



Chapter 7

The Estimation Process

Agreeing that useful estimation is of importance and that imagery probably plays a vital role within it, what exactly *is* the estimation process, and how does image building fit into it? Brenchley, 1986, and Forrester et al., 1990, both placed importance upon the formation of images, and whilst I agree with Bishop, 1989, that imaging is an intensely personal experience I would still speculate that there is a case to be made for trying to build up valid imagery in children to help them in their work. Evolving a model of the estimation process might help to clarify the issues involved.

In creating such a model of estimation I will focus on a narrow understanding of estimation referring to the actual process taking place within the child's mind, (rather than a broader model incorporating the many additional factors which affect the process) since it is with this that my study is concerned.

I would suggest from my former writing that the following is the standard teaching model followed by the main teaching schemes at Key Stage 2. Using distance as the measure under scrutiny we find:-

Handle a metre
with
Measure various distances
with
Counting skills
=

ESTIMATION

that the length of the school hall is y metres

(Although I have listed the three components in an order this is only for clarification. They are in fact constituents which can interplay and are not meant to be sequenced in time. The same is true of any later models.)

Hall, 1984, affirms such a teaching plan:-

A strategy for teaching estimation is to have students estimate, record the estimation, calculate or measure, record the result, and then determine how accurate the estimate is by comparing it with the actual measurement. The more experience a person gets in estimating first and then getting the exact answer, the better a person becomes at estimating. (p 516)

But underlying this model seem to lie two assumptions: (1) that a mental image of a metre is built up and (2) that the child has the ability to transfer this mental image to larger/smaller amounts. Yet these factors are rarely mentioned and do not appear to function in the teaching plan. This would also appear to be the case in the content of an important text book for student teachers by Williams and Shuard, 1982.

In Chapter 7 they confirm the importance of mental images in carrying out estimation, as also the ability to mentally divide a larger amount into parts the size of the imaged unit:-

The primitive way of assessing a quantity is based on a visual or manipulative estimate, comparing one whole thing with another or with an image recalled from past experience. Someone who wishes to estimate a long distance will compare it mentally with a distance that they recall, such as the length of a cricket pitch or a familiar path. They may have grown used to carrying 20 kg and will estimate a heavier mass by its feel in comparison with remembered sensation of carrying a 20 kg mass. **In other words they mentally divide the whole length or mass into parts. Measuring a continuous property depends on this awareness of a whole and its parts.** (p 271) (my emphasis)

A similar point is made later:-

Let us suppose that children want to measure the height of a house, or of a wall of the school hall. First they can estimate the height of a door in the wall, or imagine how many times a child's height would fit into the height of the wall. (p 271) (my emphasis)

Despite these references Williams and Shuard give no indication of the need to work on these two skills other than that given in the teaching plan above, of handling units and working with them on measuring tasks, the latter incorporating counting skills.

I would certainly agree that Counting skills are a vital component of the estimation teaching model. There *is* a sense in which Counting skills need no explanation for their inclusion. Without the ability to count children will only be able to directly compare two measures; for example they could say that the class book corner is "about the same size as my bedroom." They would not have the mental prowess to estimate that the length of a room is for example x metres because they have not yet learnt to count to x . However the importance of counting skills in the estimation process is not to be overlooked. Newman, 1984, showed that well developed counting skills considerably aid estimation, albeit his study was to do with numerical estimation:-

The findings concerning accuracy of estimation showed that children with fluent skills of counting are better estimators than children with less highly developed skills. (p 118)

Sikes, 1990, also writing about numerical estimation confirms this view:-

Children's ability to estimate increases with age. The reasons for this are: increased skill and strategy use in counting, experience with counting larger and larger quantities, and other varied experiences with number which provide children with more varied number sense. (p 18)

Forrester, Latham and Shire, 1990, propose a model for estimating. They outline three specific inputs into the process which are antecedent to the act, and two which are employed in the process. The antecedental elements are 'Social-discursive skills', 'Specific problem-space skills' and 'Background abilities'. The strategies involved in the actual process are 'Approximation skills' and 'Procedures'.

Forming a narrow model would exclude the antecedental elements without denying that these are of extreme importance. 'Procedures' would therefore seem to be the element holding essential significance for my own model. I will also look later at the role of approximation skills.

Forrester et al. divide 'Procedures' into three parts:- (a) Strategies (b) Judgement skills and (c) Conservation abilities.

Do we need to consider Conservation abilities? It might be expected that the average upper Key Stage 2 child, aged between 9 and 11, will have passed through the preoperational stage and therefore have developed conservation abilities. But whilst Piaget was convinced that most children would be able to conserve number by the ages of 6 or 7 this is not so for the conservation of measurement. Copeland, 1984, believes that the concepts are not developed until a later age:-

Just what are the stages through which children go as they develop an understanding of the geometry of measurement? Contrary to what might be expected, the necessary concepts are not fully developed **until approximately eleven years of age.** (p 254) (my emphasis)

According to Copeland different measures are conserved at different ages:-

Concerning linear measurement by the child, it appears that the necessary conservation concepts are achieved on the average at age seven and one-half. Measurement in its operational form is not achieved until eight or eight and one-half years of age. (p 276)

The concept of conservation or invariance as applied to volume is found to develop later than as applied to such ideas as number, length or weight.

(p 306)

Children around the age of eight begin to conserve interior volume

But it is not until eleven to twelve years of age that children can understand the idea of conservation of occupied or displaced volume.

(p 320)

It would seem that conservation abilities *will* need to be taken into consideration. The assumption that children will be inevitably conserving by nine, ten and eleven is not necessarily true. Therefore it will have to be allowed for in the teaching plan.

What exactly are the strategies and judgement skills involved in estimation? In their own study Forrester et al. found that by the age of six 31%, by the age of seven 38% and by the age of eight 64% of the children 'Imagined and counted' when working out estimations. Imagining and counting involved forming an image of a measuring unit and using this to work out their estimation by counting procedures. As children reach 9 and 10 the percentage working in this way should presumably increase. They do however add:-

Beyond suggesting that, where appropriate, children will employ 'benchmarks' usefully (and consistently), it remains unclear what is the precise relation between the children's perception of the unit, target (or the relation of one to the other) and their strategies for estimating. (p 292)

Janet Ainley, 1991, also discusses the process of estimating. She suggests that this contains the 'key features of context, purpose, limited range and rapid feedback'. Her model here refers to a wide analysis of the process and again stresses the role of context. Unfortunately for my own purposes most of the article bears little upon the actual process going on within the child's mind at the moment of estimation. It is not until she reaches her conclusion that she throws light on the content of this process:-

In the typical measurement lesson, even though estimation is nominally part of the activity, most or all of the key features I have identified are missing. There will almost certainly be little discussion of the process of estimation: it is often described as making a sensible guess, suggesting that there is no technique which can be learned. The process of estimating may involve visualizing, counting, approximate calculations, subdividing the quantity to be measured: all these are important processes in mathematics. (p 73)

Here we find highlighted 'Visualization', 'Counting', 'Approximation' and 'Subdividing the quantity to be measured'. There is considerable overlap with the Forrester model. However where Forrester et al. highlight the use of benchmarks Ainley uses counting and subdividing the quantity to be measured. Both are important and the use of benchmarks by children usually includes counting and subdividing the quantity to be measured. For example if you know that the distance from the door to the bookshelf is 2

metres and you are estimating the length of the wall, you need to subdivide the rest of the wall into units the length from door to bookshelf, then add them up and multiply by 2. This is a skill also recognised by Brenchley, 1986. She cites the case of Tim, aged 9.4 who, when explaining his own experience of estimating shows:-

the iterative aspect of measuring, the images formed and the ability to describe them and the actions performed on them.

So far then and leaving aside approximation skills for later discussion the two models have in common:-

- ♦ Visualizing/Imaging
- ♦ Counting
- ♦ Ability to use of benchmarks/subdivide units

Hildreth, 1983, also identifies similar skills when he looks at the use of strategies in estimating measures. He emphasises the complexity of the estimation process, overlooked by so many school textbooks, and amongst a list of abilities and concepts involved we find:-

a mental image of the unit that is being used in the estimation task,
the ability to perform unit iteration. (p 50)

His conclusion finds that there is a correlation between estimation ability and strategy use with perceptual ability. This bears upon the importance I am laying on the need to work on the building up of mental imagery. Whilst he finds a case for actual instruction on unit iteration and other components of estimation he makes no suggestion about what might help build up perceptual ability. What is clear though is his support for the components highlighted by both Ainley and Forrester et al.

Before we add these new components to the teaching model this is perhaps the place to discuss the sorts of images which may be summoned by the different measures and to acknowledge that there may well be distinctive differences. These differences may in turn affect the ease of imagery for the estimator, or bring with them unique problems. My use of the term 'imagery' has been broad and encompasses a variety of inner experiences - visual, kinaesthetic, auditory, dynamic etc.. We can see how different measures might summon different kinds of mental image. For example length measures often summon very visual images. Pictures spring readily to mind when we consider the metre or centimetre, (or perhaps the yard, foot and inch for those at school before the metric system was introduced). In contrast images of weights may involve a combination of visual and kinaesthetic imagery and a sense of feel built up from past experience. For

capacity estimation a static visual image may need to be manipulated and changed into a very different three dimensional shape.

Time estimates too carry their own particular body of images. Estimating a time may involve working through dynamic images within one's own mind, for the purposes of comparison. Or a time image may arise from a very strong association - my mental image of 10.30 a.m. is always of a playground full of children. Time brings its own problems, we are all aware of the truth of the sayings 'time stood still' and 'time flies'. It is important to recognise that different measures may call for different kinds of images.

With these last points in mind we are now ready to add all the new components we have specified to our previous teaching model. Adding to it we get:-

Handle a metre

with

Build image of metre

with

Measure various units

with

Ability to use benchmarks/subdivide units

with

Counting skills

with

Conservation abilities

=

ESTIMATION

that the length of the school hall is y metres

Before completing the teaching plan we need to consider one other input mentioned by Forrester et al., approximation skills. Ainley too made reference to the use of approximate calculations in the estimation process.

There has been debate as to whether there is a clear distinction between estimation and approximation at all. This distinction is discussed by Sowder, 1992. She reaffirms the more generally held view which does differentiate between the two and she quotes from Thompson.

Thompson (1979) calls an estimate an educated guess, usually made in the context of the number of objects in a collection, the result of a numerical computation or the measure of an object. It is often difficult to assess the magnitude of error in an estimate. Approximating, on the other hand, is attempting to close in on a target value. (p 373)

Hall, 1984, uses Thompson's distinction in his own writing and adds.

Approximating is finding a result precise enough for a specific purpose. (p 517)

These distinctions are helpful and it is easy to see how approximation, as a separate skill, could be used independently of estimation. For example using pen and paper to work out 3 divided by 14 to three decimal places is giving an approximate answer. Estimation is not involved here, simply the ability to divide by two figure numbers. However I do not believe that the obverse is true. I think it will be seen that approximation is nearly always involved in the estimation process, that estimation is estimation of approximate rather than exact values.

In discussion of this point we need to make a distinction between approximation in estimation and the inevitable approximation that is found within measurement, due to its inexact nature. As Ainley writes

Making any measurement involves some degree of approximation. (p 75)

Hall emphasises the same point

All measurements are approximations, that is, they are close to the target value but not exact. (p 517)

The OU PME 233, 1980, spells this out more fully.

... it is impossible to measure most things exactly. This is partly dependent upon the tools you use and partly on your ability to read off the

measurement to more than a certain accuracy. Even with the aid of such devices as a magnifying glass and vernier scale there is a limit to the smallest subdivision you can distinguish. There is a limit to the sensitivity of all measuring devices, however sophisticated, and so there will always be a margin of error. (p 45)

But what of approximation within estimation? Unfortunately Forrester et al. in no way elaborate upon what they mean by approximation within this context. Ainley, by focussing on 'approximate calculations' gives more idea and Hall's definition also helps. Approximations suitable for specific purposes are used in making estimations. Some examples help to clarify what is meant. When asked to estimate the area of a rectangular room many people would estimate the length and width by approximating them to whole or whole and half numbers, thus making the estimation manageable. They would then multiply the two estimates to get their estimated area. Approximation has been intricately involved in this estimation process. Another example might be when you ask someone to estimate the amount of time it would take to drive to Newcastle from London and they give you the answer in hours and half hours rather than in more detailed hours, minutes and seconds, for an approximate estimate is what is needed. Approximation has taken place within the estimation.

So making an estimation will generally involve giving an approximate value not an exact value and the degree of approximation will find relevance in the purpose of the estimation. PME 233:-

Different purposes require different accuracy. Sometimes it is better to overestimate, as with measuring water into the kettle for a cup of coffee. Other times it is better to underestimate, as with the amount of petrol left in the petrol tank. (p 10)

So we *will* need to add approximation to the process of estimation taking place within the child's head.

A teaching model now contains:-

Handle a metre

with

Build image of metre

with

Measure various distances

with

Ability to use benchmarks/subdivide units

with

Counting skills

with Conservation abilities

with

Approximation

=

ESTIMATION

that the length of the school hall is y metres.

Effective teaching results in the children integrating these components to produce an overall sense of 'metre'. All of these components will need input and teaching in the primary classroom. A full consideration of the model is however beyond the scope of this thesis. My empirical studies which follow concentrate only on the role of imagery within the estimation process. Whereas the typical school plan assumes that the building up of a meaningful image of unit happens automatically as other components of the plan are put into action I would like to disagree and surmise that the building of such an image may need specific teaching and that such input will result in an increase in accurate measurement estimation.

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IMAGERY AND ESTIMATION

'We have paid too little attention to the role of images
in mathematical understanding.' (Higginson, 1982)

Measurements frequently summon vivid
images. Could this have consequences for
the teaching of measurement estimation
skills to junior aged children?



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Volume III of III

Chapter Eight

Using Imagery in Estimation with Ten Year Olds
- an Initial Foray



Chapter 8

Using imagery in estimation with ten year olds - an initial foray.

How might a deliberate teaching effort aimed at building up images in children's minds affect their estimation work? I did an initial foray into this area of research working with the same group of children I worked with in Chapter 6. The only difference in the group was that the children were by then six months older and there were three fewer participants, so that the group now numbered twenty nine.

The exercises involved actually encouraging, and at times requesting the children to image in their minds as they made their estimations. The children were helped in this by discussion about images and what they might be, such as pictures in the mind.

Over a two week period we worked on estimation in time, capacity, length and area. I felt that it was important to give the children sufficient time to build up an image of the unit in which I wanted them to estimate. So at the start of each session considerable time was spent on image building. How

could I ask the children to form an image of a unit in order to estimate if they had no valid image of the unit already? It might be like asking a non-French speaker to estimate how many 'timbres' would cover an exercise book. The task would have no meaning for him as he would have no visual image of a 'timbre'. He could as well estimate 5 or 55 or 555! However once he knows that it is a postage stamp then the whole exercise has validity. Similarly not to give the children a valid image of the entity in which they were being asked to estimate, would nullify the whole process.

The first session opened as follows:-

Teacher: Do you remember when I asked you to do some estimations for me two terms ago? Things like - How much water does the sink hold? How long would it take to walk to the infant school? How tall is your Mum?

Responses in the affirmative.

Teacher: We are going to do some similar work because you gave such interesting answers. But we're going to do them a bit differently this time. Today we are just going to do *time* estimates. Let's start by finding out exactly how long a second is.

Here followed a variety of exercises using a watch so that the children should be reacquainted with the length of seconds and minutes. These exercises were already familiar to the class since they were sometimes done as class games at the end of a day. One example consisted of putting their heads down on their desks and lifting them up when they felt half a minute had passed. Another consisted of timing a child as they walked down to the third year class and back.

Once these exercises were completed I continued.

Teacher: You can give and estimate in seconds or in minutes or in both, and don't forget how many seconds there are in a minute.

This brought forth a response from some children.

Teacher: Question A. Close your eyes and imagine yourself walking out of this room, down the stairs to the year 6 area, through the TV area, past the Headteacher's office to the staffroom door. Turn around and imagine yourself *walking* all the way back - going over in your mind all your steps.

Right, now do that all again in your mind but this time work out how long you think you are taking and then when you have finished open your eyes and write down your estimate of how long you think it would take you to get to the staffroom and back.

The children then carried out the exercise.

Teacher: Now let's try it out. Two of you can walk to the staffroom and back and we will time you.

Then followed three more tasks with the children closing their eyes and carrying out the activity in their minds. They next wrote down their estimate of how long they thought it would take them to complete it. The actual task was then carried out by two of the class while the rest of us timed them. The tasks completed were the following.

Question B. How long would it take you to go to the hut and back?

Question C. How long would it take you to jump up really tall 10 times?

Question D. How long would it take you to go from the front of the room to the sink, wash your hands, dry them, put the towel in the bin and come back to your starting place?

The next day length was considered.

Teacher: Today we're going to do some estimations to do with length.

Let's have a really good look at this metre stick first. Look really hard at it and then close your eyes and see if you can still see it in your mind's eye. Then open them and check what you saw. Repeat this several times if you want to.

The children carried out the exercise, and plenty of time was allowed for it.

Teacher: Question A. Close your eyes and imagine the metre stick.

Now open your eyes and look at the blackboard. Put the imaged metre stick next to it and measure the blackboard with this imaged metre stick. When you have finished doing that write down your estimate.

The children completed the exercise and then two of the children actually measured the board for us. The measuring took place after each task as before. In a similar way the children did the following:-

Question B. Estimate the distance from the classroom window to the cupboard using metres.

Question C. Estimate the length of the school hall using metres.

Question D. Estimate the length of the corridor from year 5's wall to the top of the stairs by year 4.

On two following days exercises were carried out in the estimation of capacity and area. Similar time was spent building up the image of the units used before the children gave their estimates and the children were again encouraged to image the process in their minds as they undertook the estimation.

When the work was finally finished I encouraged them to comment upon the tasks. The unanimous response was one of enjoyment. A majority said that closing their eyes and actually seeing the measuring unit had made them feel more confident in giving their estimates. They didn't feel that they were guessing so much.

This exercise was very limited in what it set out to do but a look at the results is interesting. Did the children simply gain confidence and enjoy the exercise or might there be some indication of a greater accuracy in their estimations?

The results of the work are shown in tables 2 and 3. The tables differ from table 1 (see Chapter 6) in the way in which they are set out. They are laid out under the four measurement headings of length, time, area and capacity and there are of course more questions under each heading.

Table 2

Child	Time Questions				Length Questions			
	1	2	3	4	1	2	3	4
1	Sr	Sr	Sr	Sr	Sr	Sr	Sr	Su
2	Sr	Sr	Sr	Sr	Sr	Su	Su	Su
3	Sr	Sr	Sp	Sr	Sr	Sr	Sr	Sr
4	B	Su	Sr	Sr	Sr	Sr	Su	Sr
5	Su	Sr	Sr	Su	Sr	Sr	Sr	Sr
6	Sr	Sr	Sr	Sr	Sr	Sr	Sr	Su
7	Sr	Su	Sr	Sr	Sr	Sr	Sr	Sr
8	Su	Sr	Sr	Sr	Sr	Sr	Su	Sr
9	Su	Su	Su	Sr	Sr	Sr	Sr	Sr
10	Su	Su	Sr	Su	Sr	Sr	Su	Sr
11	Sr	Su	Su	Su	Sr	Sr	Sr	Su
12	Su	Sr	Su	Sr	Sr	Sr	Sr	Su
13	Su	Sr	Su	Sr	Sr	Sr	Sr	Su
14	Sr	Su	Sr	Sr	Su	Sr	Sr	Sr
15	Su	Sr	Sr	Sr	Su	Su	Sr	Sr
16	Sr	Su	Sr	Su	Sr	Su	Sr	Su
17	Sr	Sr	Sr	Su	Su	Sr	Sr	Su
18	Sr	Sr	Sr	Sr	Sr	Sr	Sr	Sr
19	Su	Su	Sr	Sr	Sr	Sr	Sr	Su
20	Sr	Sr	Sr	Sr	Sr	Su	Sr	Su
21	Su	Su	Sr	Sr	Sr	Sr	Sr	Su
22	Sr	Su	Sr	Sr	Sr	Sr	Sr	Su
23	Sr	Sr	Sr	Sr	Sr	Sr	Sr	Su
24	Su	Su	Su	Sr	Sr	Sr	Sr	Su
25	Su	Sr	Su	Sr	Sr	Sr	Sr	Sr
26	Su	Sr	Su	Sr	Sr	Su	Sr	Su
27	Su	Su	Sr	Sr	Sr	Su	Sr	Su
28	Sr	Su	Sr	Sr	Su	Su	Su	Sr
29	Su	Su	Sr	Su	Su	Su	Sr	Su

Table 3

Child	Capacity Questions				Area Questions			
	1	2	3	4	1	2	3	4
1	Su	Sr	Su	Sr	Su	Sr	Su	Sr
2	Su	Sr	Sr	Su	Sr	Sr	Su	Su
3	Su	Sr	Su	Sr	Su	Su	Sr	Sr
4	Su	Su	Su	Su	Su	Sr	Sr	Sr
5	Su	Sr	Su	Su	Sr	Sr	Sr	Sr
6	Su	Su	Su	Su	Su	Su	Su	Su
7	Su	Su	Su	Sr	Su	Su	Su	Su
8	Su	Su	Sr	Sr	Su	Su	Sr	Sr
9	Su	Sr	Sr	Su	Su	Su	Sr	Sr
10	Su	Su	Sr	Sr	Sr	Sr	Sr	Sr
11	Su	Su	Su	Sr	Sr	Sr	Su	Su
12	Su	Su	Sr	Su	Su	Su	Su	Sr
13	Su	Sr	Sr	Su	Su	Sr	Sr	Su
14	Su	Sr	Su	Sr	Sr	Sr	Sr	Sr
15	Sr	Su	Su	Sr	Sr	Sr	Su	Sr
16	Su	Sr	Su	Su	Su	Su	Sr	Su
17	Su	Sr	Su	Su	Sr	Su	Sr	Sr
18	Sr	Su	Sr	Sr	Sr	Su	Su	Su
19	Su	Sr	Sr	Su	Su	Su	Su	Su
20	Sr	Sr	Su	Su	Sr	Su	Su	Sr
21	Su	Sr	Sr	Su	Su	Su	Sr	Su
22	Su	Sr	Sr	Sr	Sr	Sr	Su	Su
23	Sr	Su	Su	Su	Sr	Sr	Sr	Sr
24	Sr	Sr	Su	Sr	Sr	Su	Su	Sr
25	Sr	Su	Su	Sr	Sr	Su	Sr	Su
26	Su	Sr	Su	Su	Sr	Sr	Sr	Su
27	Su	Su	Su	Su	Su	Su	Sr	Sr
28	Sr	Su	Su	Su	Su	Sr	Sr	Sr
29	Su	Su	Su	Sr	Su	Su	Su	Su

I have also chosen to categorise the answers in only three ways rather than the thirteen of before. This is because I was asking different questions of the exercise. I have only allowed for answers in standard units and simply for reasonable or unreasonable estimates. The only exception to this was an allowance for a blank answer in case the children felt unable to give an estimate. Looking at the results is therefore less complex. Here are the three categories with their codes:-

Sr - stands for a reasonable guess using standard units.

Su- stands for an unreasonable guess using standard units.

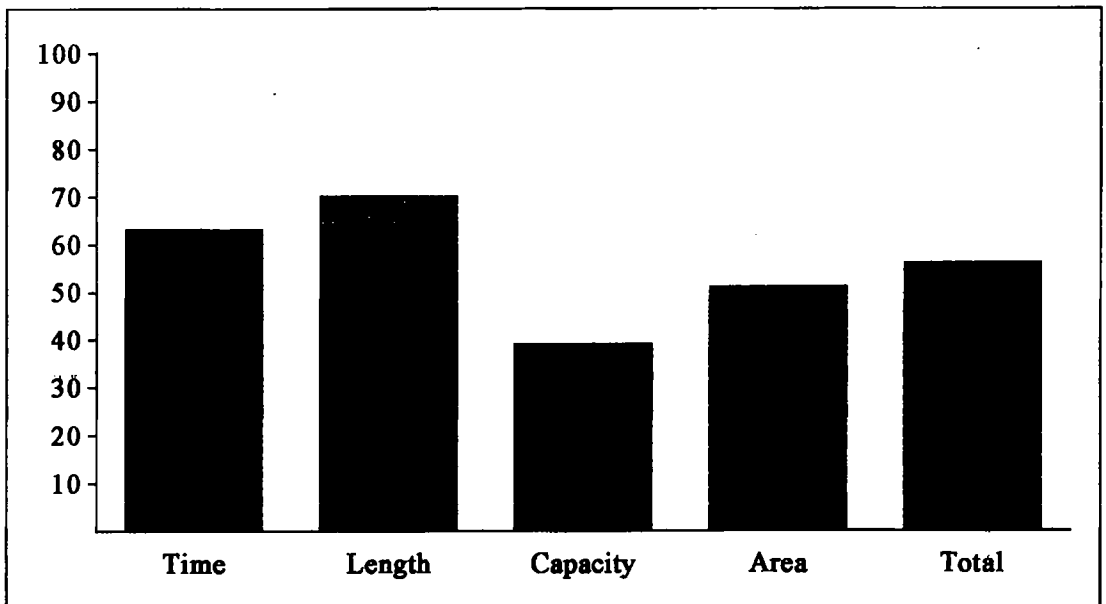
B - stands for a blank answer.

Looking at the children's responses show some interesting trends. Out of a total of 464 answers only one was left as a blank. Does this indicate a growing confidence in the children to estimate? Is this due to the imagery which they were encouraged to summon?

On the following page are the figures showing the percentage of reasonable results:-

REASONABLE RESULTS

	Reasonable results	No of responses	%
Time	73	116	63
Length	81	116	70
Capacity	45	116	39
Area	59	116	51
Total	258	464	56



These results cannot be compared in any direct way with my previous study. This is because in the previous instance the children were allowed

complete freedom in the way in which they chose to record their answers. They could use direct comparison or non-standard units in addition to standard units. This was not allowed the second time. In the first study I also looked at five instead of four fields of measure and asked less questions in each field, only two as opposed to four. What we *can* get however, is some overall feel as to how the children are now performing.

At first sight the overall results are encouraging. The total of reasonable answers using standard units was over half. There does appear to be considerable discrepancy in accuracy between the four measures used. Whilst length estimates resulted in nearly threequarters being reasonable those of capacity gave only just over a third. Time came out well over a half and area just less than this. Why should this be? And is it due in part to children's ease of imagery in e.g. length rather than capacity? In the first study most of the answers to capacity were given in non-standard units or by direct comparison. Perhaps they found it much harder to summon accurate images in capacity using standard units? But it is perhaps not surprising that there is a discrepancy in the accuracy of different types of measures. Unlike length both area and capacity occupy more than one dimension. The same area can be created by rectangles of different sizes. Capacity, in occupying three dimensions and being fluid can remain

constant, yet take on a whole variety of shapes. It is much more difficult to compare areas or capacities mentally than it is length in one dimension. Therefore a greater inaccuracy might be expected.

Yet the general impression I gained from working with the children was that these short exercises had helped to give them both greater confidence in their work and also led to a greater accuracy in their estimations. I felt it would be worthwhile to pursue some more in-depth work with other children to see if I could gain further indications that an increased use of imagery in estimation could increase accuracy.

Chapter Nine

Can an Increased Use of Imagery in Estimation
Increase Accuracy?



Can an increased use of imagery in estimation

increase accuracy?

I next undertook some work with KS2 children to see what might result from a programme composed of tasks designed to deliberately invoke their use of imagery when making estimations.

Programme Outline:-

Initially six year five children in a local Tunbridge Wells primary school were involved. They were chosen for the project by their classroom teacher; not for their mathematical ability but only insofar as they represented a cross section of abilities. Three were boys and three were girls.

The areas of measurement in which I worked were narrow as I felt that this would enable the children to do meaningful and concentrated amounts of work on building up images, albeit in a small sphere of measurement. The measures I chose were the centimetre for distance work and the litre for capacity. My choices were determined in part by the measurement areas

where previous children in my study had demonstrated the poorest and best results in accurate estimation, in order to provide contrast. They had proved most competent in distance estimation and least competent in capacity estimation. For distance estimation I chose to work with the centimetre rather than the metre, although the latter would also have been appropriate. My reason for this choice was to provide as great a variety of materials for the children to work with as I could; for example I could use the daily objects that lie on their desks like pencils and rubbers. For work on capacity I chose the litre because I felt that no other measure was really appropriate; I needed to use metric measures without using half measures and millilitres were too small.

On my initial visit to the school I took a questionnaire with which to test the children's estimation accuracy. Eight questions involved estimating in centimetres and eight in litres. (see Appendix 3)

After the children had filled in the questionnaire I asked each child, individually and away from the group, how they had tried to work out the results. These were their responses:-

Sarah 9.11 'I just guessed'

Katie 9.4 'by looking' and 'measuring up'

Edik 10.0 'just guessed'

Luke 9.10 'I don't know'

Hannah 9.6 and Jack 9.2 were unable to verbalise their thoughts but just knew that they did it.

Katie's answer was interesting. Although she didn't mention an image, or picture in her mind she said she was 'measuring up'. This indicates that she was using some sort of image with which to measure up. It would have been useful to have questioned her further.

I then produced the materials I had brought for them to work with, enclosed in two containers. Container A held a variety of objects appropriate for centimetre measurement, for example:- paper clips, elastic bands, pieces of paper, pencils and pieces of string. I had also placed within the container some red centimetre squares drawn and coloured flat on paper.

The children next discussed what they imagined a centimetre to be and we used rulers to check these. The most common one was the width of their finger nail. We also looked at the red squares. After discussion I then asked the children to close their eyes and imagine a centimetre measure,

indicating that they could use the red squares we had been looking at or any other approximate measure they had found such as a finger nail. They then opened their eyes, chose one of the objects in the container and I encouraged them to think about how many of their centimetre images they could fit along it. We then measured the object and talked about the results. This was repeated with all the other objects.

Container B held a variety of capacity measures such as lemonade bottles and plastic boxes. We talked about the litre measure and all the children decided that the only litre measure they were really familiar with was that used to hold orange juice. There was an orange juice carton in B and I then encouraged the children to close their eyes and see the carton in their imagination. When their eyes had been opened I then held up one of the other objects and asked them to try to work out how many cartons of orange would fit in it. The children then checked out their estimations by filling up the containers using water from the classroom tap.

During the following week the class teacher let each child work independently with the materials two or three times. Each time the child was to close their eyes and image their chosen measure before attempting to estimate, and then afterwards check their estimate by practical work.

I returned to the school a week later, changed over all the materials, and met the children again, taking them through the same procedure of imaging and then estimating with their image, but using the new materials. They were then left to repeat the procedures during the next seven days.

During the third week I again repeated the process and changed the materials over for the children, leaving them one final week to work on the materials.

Analysing the Estimation Test

Meanwhile I had looked at the estimation tests which the children had completed during our initial meeting. For my present purposes I felt I needed a more structured way to compare the different estimates given. Therefore in looking at the results I made my assessment of the estimates more formal. I calculated the relative error of each estimate in the following way:-

The difference between the estimate and the correct measurement
divided by the correct measurement = the relative error.

I was prepared to accept as a very good estimate a relative error of 0 - 10%. This meant a 16cm pencil would need to be estimated with less than 2cm out either way, a difficult task for a nine year old. I accepted as good

a relative error of 11 - 20% . So for a good estimate a 4cm rubber would need to be within 1cm either way. I gave as satisfactory a relative error of 21 - 30%, which would have meant estimating that same rubber as 3cm or 5cm, and as barely satisfactory a relative error of 31 - 40% making the rubber as small as 2.4 cm or as large as 5.6 cm. Anything else I considered to be an inappropriate estimation. It may seem generous to allow for relative errors of up to 40% but when dealing with small amounts the errors are magnified. For example an estimate of 1cm for a 2cm strip of paper would give a relative error of 50%. An estimate of $\frac{1}{2}$ litre on a mug containing 300ml would give a relative error of 67%. Both would be classified as inappropriate estimations.

The results of the tests are shown in tables 4 and 5. Table 4 shows relative errors in the centimetre estimation test. There are a total of 48 responses. Of these only 3 give a very good estimate. Five give a good response. Combining both very good and good results give us 16%. Into the satisfactory category come 13% and into the barely satisfactory category 10%. This leaves well over half of the answers showing inappropriate estimation..

RELATIVE ERRORS IN CENTIMETRE ESTIMATION TEST

		Questions							
		1	2	3	4	5	6	7	8
Child	1	83%	114%	69%	29%	67%	70%	43%	33%
Child	2	15%	40%	33%	43%	42%	45%	31%	43%
Child	3	42%	5%	49%	43%	53%	76%	66%	43%
Child	4	15%	38%	28%	29%	7%	15%	31%	14%
Child	5	76%	81%	79%	79%	81%	82%	83%	79%
Child	6	2%	29%	49%	29%	44%	39%	20%	52%

Table 4

RELATIVE ERRORS IN LITRE ESTIMATION TEST

		Questions							
		1	2	3	4	5	6	7	8
Child	1	50%	48%	67%	68%	233%	23%	39%	576%
Child	2	40%	43%	56%	3%	11%	4%	39%	150%
Child	3	70%	83%	89%	68%	11%	90%	92%	233%
Child	4	50%	57%	67%	35%	44%	62%	55%	17%
Child	5	70%	70%	94%	84%	72%	80%	92%	44%
Child	6	0%	57%	44%	35%	11%	23%	39%	67%

Table 5

Turning to Table 5 we see the results of the litre estimation test. Once again 3 answers are very good estimations. However less show good answers, giving a total for very good and good estimations of 14%. Into the satisfactory category come 4% of the answers and into the barely satisfactory 10%. This leaves a total of nearly three quarters of all answers as inappropriate estimation.

With a total of 96 answered questions the results from the two tables can be combined:-

Very Good	6	6%
Good	9	9%
Satisfactory	8	8%
Barely satisfactory	10	10%
Inappropriate	63	67%

These results indicated that there would be plenty of room for improvement in the children's estimation skills.

During my final visit to the school I met with the children and gave them the same estimation test as before. I also talked with them about what they were doing. The taped transcripts follow but first I shall look at the new

test results. Unfortunately one of the children was away from school on long term leave so I was unable to find out how he would have performed on the test. He was child 5, all of whose estimates had been inappropriate on the first test so it would have been particularly interesting to see how he now approached the work.

The new results can be seen in tables 6 and 7. Since only five children were involved in the testing only forty questions were answered in either test. Firstly I looked at the centimetre estimation. 15 answers were very good, 4 were good, giving a total of good and very good of nearly half. There were now 8 satisfactory answers and 6 barely satisfactory answers, leaving 7 answers as inappropriate. This compares very favourably with the estimates given by the children before they started working on the project.

Looking at the litre test, very good answers were 6 and good answers made up one fifth. These also show an improvement on the previous estimates. Satisfactory answers accounted for 7 more responses and barely satisfactory for 6 more, leaving as inappropriate responses 13, roughly one third. This compares with a previous total of nearly three quarters.

After Imagery Work

RELATIVE ERRORS IN CENTIMETRE ESTIMATION TEST

		Questions							
		1	2	3	4	5	6	7	8
Child	1	133%	138%	0%	78%	27%	139%	18%	43%
Child	2	15%	10%	28%	7%	26%	3%	20%	33%
Child	3	5%	5%	28%	7%	35%	33%	37%	43%
Child	4	10%	24%	15%	7%	7%	3%	9%	24%
Child	5								
Child	6	42%	43%	28%	36%	26%	3%	9%	5%

Table 6

RELATIVE ERRORS IN LITRE ESTIMATION TEST

		Questions							
		1	2	3	4	5	6	7	8
Child	1	50%	65%	44%	3%	233%	59%	21%	233%
Child	2	15%	10%	28%	7%	26%	3%	20%	33%
Child	3	30%	35%	33%	18%	11%	27%	39%	67%
Child	4	20%	13%	44%	125%	67%	23%	9%	17%
Child	5								
Child	6	0%	22%	44%	32%	233%	15%	39%	67%

Table 7

Putting the two sets of results together we find:-

Very Good	21	26%
Good	12	15%
Satisfactory	15	19%
Barely Satisfactory	12	15%
Inappropriate	20	25%

At first sight the new figures look very encouraging and would seem to indicate that the work with which the children had been involved had helped to make their estimations far more accurate. This work had involved deliberate input on the building up of images of the centimetre and the litre. There are also other interesting features of the data. One of these was to show that these children as well as my previous group found capacity estimation more difficult than distance estimation. Certainly these children had greater difficulty in working with the litre measure than with the centimetre measure. The initial inappropriate estimates for litre measurement were higher than those using the centimetre. After the work on imagery the gap was still there and had slightly widened, although there was an overall improvement.

It is also interesting to look at how individual children worked since the results were in no way uniform. One or two children showed a marked improvement in accuracy whilst some other results were puzzling.

Child 1

Initially Child 1 performed very poorly on the test. Overall he had 2 satisfactory answers and 2 barely satisfactory answers, with none reaching a good category. After work on imagery he managed 2 very good estimates, 1 good estimate and 2 satisfactory estimates. This indicates that he was beginning to work with greater accuracy but I would speculate that he would need considerably more input before he was achieving at a confident level as the majority of his answers were still inappropriate.

Child 2

Before imagery work Child 2 managed 2 very good estimates, 2 good estimates and 3 satisfactory estimates, leaving more than half of his answers as inappropriate. After the project he achieved 6 very good estimates, 4 good estimates, 4 satisfactory estimates and 2 barely satisfactory estimates. This left none of his answers as inappropriate. This child had obviously radically improved his ability to estimate with a litre and a centimetre.

Child 3

Child 3 had little success with accurate measurement in the initial test. She achieved one very good result and one good result, all the rest were inappropriate estimations with the litre estimations being particularly poor. After the input on image building and estimation work her accuracy had vastly improved. 3 results were now very good, 2 were good, 3 satisfactory and 6 barely satisfactory. This only leaves 2 results as showing inappropriate estimation and the test showed a marked improvement in accurate estimation.

Child 4

In many ways Child 4 had performed with the greatest accuracy in the initial test. She had 1 very good result, 4 good results, 2 satisfactory results and 3 barely satisfactory results. Following the imagery work she still performed well and with greater accuracy. 6, as opposed to 1 very good result, 4 others being good and 3 satisfactory. There were now 3 as opposed to 6 inappropriate estimations. Her litre estimation was weaker than her work on centimetres. One result, with an error of 125% seemed inconsistent with the rest of her estimation work.

Child 6

Child 6 had on initial testing 2 very good estimates, 2 good estimates, 2 satisfactory estimates and 4 barely satisfactory results. This leaves 6, or just over one third as inappropriate estimates. In the second test her estimates were categorised in the following way; 4 were very good, 1 was good, 3 were satisfactory and 4 were barely satisfactory, leaving 4, or a quarter, as inappropriate. Obviously these figures show some improvement but there are some surprising findings in Child 6's results. She shows considerable improvement in her centimetre estimates but if you look closely at the litre estimates she performs no better at all. She has improved on some questions but has done worse on others.

Discussion with the children

I spoke to the children in two smaller groups at the end of the project.

Below follows some of their discussion:-

Luke: (explaining to Katie how to estimate the length of the tape recorder)

I would think of a centimetre square and then move it along the tape recorder.

Katie: (explaining to Luke how to measure the length of the photo frame)

I would look at the centimetre square.

Teacher (interrupting): How would you look at it?

Katie: Close your eyes.

Teacher: So you see it in your head do you?

Katie: Yea. And then put it along the side.

Teacher: Can you think of a really hard question for Luke to answer and
Luke a hard question for you?

Luke: On top of the window.

Teacher: Now why is that so difficult?

Luke: Because there are millions of centimetres.

Teacher: So what would be more sensible to estimate it in?

Luke: In metres.

Teacher: So how would she imagine it in metres then?

Luke: Think of a metre stick in her head and then put it along the wall.

Katie: (asking Luke) How many feet do you think this classroom is?

Teacher: How would he work that one out do you think?

Luke: I'd think of myself because I'm four and something foot.

Teacher: Can you think of a difficult one for Katie to do in litres?

Luke: How many litres would fit in this room?

Katie: I've got a feeling you might have to use gallons there Luke.

Teacher: Have you got a picture of a gallon then?

Luke: I've got a gallon watering can at home.

Edik: (explaining how he would estimate to Sarah and Hannah) Picture the square in your mind and then move it along the line.

Teacher: Sarah can you explain to Edik and Hannah how you would estimate this?

Sarah: A centimetre is nearly the same size as my finger so I'd use my finger.

Teacher:(asking Hannah who explained her process by using the red square) So is the red square a picture in your mind, is it fixed in your mind?

Hannah: Yes

How much can we learn from the children's conversation? Firstly that most of them are more confident in describing the processes they are using to estimate than they were originally. Next that they are now speaking in terms of images and appear to be using them to help in the tasks. Luke and Katie used images when talking about estimating in gallons and metres as well as in centimetres and litres. They were therefore applying their knowledge and using images in a wider context. However it is important to remember that this is the process they were *asked* to use and in some ways their conversation mirrors the exercises they undertook. It would be interesting to visit the same children in a few years time and see how their

estimation skills have developed and whether they have been built on imagery work or not.

Several weaknesses of the study emerge. Use of the red square could have been misleading. Although it appeared to work for these children, the square is an area measure and could have caused confusion, a different choice for help with image building of the one centimetre would have been more appropriate. I also think that capacity, with its three dimensional and fluid properties, would probably have benefited from a more intensive input than length, instead of being treated to the same programme. Finally we also need to recognise that some improvement in accuracy may have been due to a more regular practice in estimation, not simply to the image building exercises.

Chapter Ten

Imagery and Estimation -
Drawing Together the Threads



Imagery and Estimation -

Drawing together the threads

This study has sought to emphasize the vital link between imagery and measurement estimation, and hence the significance of the role of imagery within estimation work. Measures can often summon vivid images; '100 yards' may be etched on the memory as a school sports field, a 'load' of timber may be seen in the mind's eye on the back of a cart or '4oz' of boiled sweets envisaged in a paper bag one is holding. An historical overview of measure words has revealed an image rich vocabulary. Yet despite this and despite the fact that so many adult measure-related images have their base in childhood memories I feel that the full significance of images within measurement work has been under utilised by educators. This may not be true for other areas of mathematical understanding. The importance of using images within mathematics teaching has been recognised now for many years and the writings of Boole, Gattegno, Presmeg and many others have emphasized and enlarged upon this meaningful issue.

My theme has narrowed from a look at measurement in general to focus on the estimation skills of junior aged children. Estimation skills are of key importance in measurement activities and are given their place within the National Curriculum; yet it is an area of work where children can experience difficulty and lack accuracy. This is a view supported both by educational research and classroom practice. My work has sought to show how emphasizing the vital role played by imagery in measurement estimation can help children to improve this accuracy

This fundamental role of imagery within estimation work has been recognised by modern educators. Despite the difficulty posed by imaging being intrinsically such an intensely personal experience, researchers do stress its central importance in enabling children to make meaningful measurement estimations. It is when we look at what is done with this knowledge that a gap between theory and practice appears. In the teaching schemes I examined I found only little evidence of work aimed specifically at building up children's images. Images are required in estimation but there seem to be assumptions made that with sufficient practice in estimation and measurement tasks useful images will naturally emerge. My thesis has taken a different turn. I would like to surmise from my work that these on their own may not necessarily be enough to help the child

estimate accurately, and that *estimation work aimed SPECIFICALLY at the building up of images of particular measures in children's minds can be valuable in helping them to make more accurate estimates.*

Clearly more work needs to be done with children in this field. I worked predominantly within the realms of length and capacity. Other types of measurement such as time, weight and area need to have studies conducted. My work with a small group of children also had limitations of its own, which I outlined in Chapter 9; there is scope for considerably more practical work in this sphere.

In addition to my tentative findings questions emerge naturally from my work. Firstly we can ask whether, once they have acquired the skill of using images more confidently in their estimations, children will subsequently transfer this skill to other types of measures; we saw Luke briefly doing this in his work with me. If this were to be the case in general then the image building exercises undertaken would have even more value in helping the children with their future work.

- ♦ Once children have acquired the skill of using an image of one particular measure confidently in their estimations, will they

subsequently transfer this skill to their estimation work in other measures?

My study has also, perhaps obviously, shown that different children responded to different amounts of input. How much time might be allotted to this type of work by the classroom teacher? Would time spent in the primary years prove well spent in terms of later understanding?

- ♦ How might image building exercises be timetabled into mathematics curriculum planning?
- ♦ Do some children benefit more than others in their estimation work from image building exercises?

My work also leads me to pose further questions which could usefully be answered by future research:-

- ♦ Does an increased use of imagery in estimation work encourage confidence in the making of estimates?
- ♦ Is children's ease of imagery better in some measures work than in others, and if so why?

- ♦ Do certain standard units conjure up images in children's minds more readily than other standard units do?

Answers to these questions would help to inform educators in their work.

'We have paid too little attention to the role of images in mathematical understanding', Higginson. This was written in 1982 and is true for measurement estimation in 1996. An increasing awareness of the role of images in our mathematical curriculum can only lead to greater confidence and achievement for children. This awareness needs to be passed on to the classroom teacher.

Appendix



QUESTIONNAIRE

AGE ☐ SEX ☐ MATHS QUALIFICATIONS _____

ATTITUDE TO MATHS _____

Relate in pictures, diagrams or words what you see or think about when you have to quantify the following:

<i>For example you might envisage the following in one of these ways:-</i>			
half a dozen	111111	6	000 000
kilogram	large bag of apples	1kg weight	picture

100 yards
.....
1 inch
.....
100 miles
.....

1000 miles
.....

1 acre
.....

100 square miles.....
.....

1 square yard
.....

1 oz
.....

1lb
.....

10 stone
.....

1 ton
.....

1 pint
.....

1 gallon
.....

1 litre
.....

1 score
.....

a century
.....

a millennium
.....

1 year
.....

1 second
.....

1 week
.....

8 a.m.
.....

midnight
.....

ANY COMMENTS

.....
.....
.....
.....
.....
.....
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.....
.....
.....

CHILDREN'S ORAL QUESTIONNAIRE ON ESTIMATION

**How big is this room?
How did you get this answer?**

**How much water could you fit in the sink?
How did you get this answer?**

**How tall is your Mum?
How did you get this answer?**

**How far is it to the staff room?
How did you get this answer?**

**How heavy are you?
How did you get this answer?**

**How long would it take to walk to the infant school?
How did you get this answer?**

**How big is the hall?
How did you get this answer?**

**How much water can you get in a school mug?
How did you get this answer?**

**How long would it take you to write out the alphabet three times?
How did you get this answer?**

**How heavy is your reading book?
How did you get this answer?**

ESTIMATION ORAL TEST

NAME

D.O.B.

How many centimetres in:-

A piece of A4 - vertical side?

This pair of scissors?

A piece of A4 - horizontal side?

This pencil?

This stapler?

A small pritt stick?

This glasses case?

This picture frame?

How many litres in:-

This bucket?

This vase?

This watering can?

This plastic container?

This washing up bowl?

This school tray?

This cake tin?

This mug?

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